Avoidance of Oversampling for Adaptive Linear MMSE Receivers in CDMA Systems

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***Abstract*—An adaptive linear MMSE receiver for CDMA systems is introduced; subsequently, the conjecture about the avoidance of oversampling is addressed. Simulation results highlight a trade-off between performance degradation during application of conventional *chip-duration-spaced* sampling with selected signature sequences and an increased cost of complexity for oversampling implementation. In addition, the suggestion on the signature sequence design for CDMA systems using the chip-duration-spaced sampling with the adaptive linear MMSE receiver is also given.**

***Index Terms*—Adaptive filters, Code division multiaccess, FIR digital filters, Multiuser channels.**

I. Introduction

Code-division multiple-access (CDMA) is a multiplexing technique in which several independent users access a common communications channel by modulating their information input with their unique signature sequences. A lot of receiver structures have been investigated [1]. One receiver structure "adaptive linear MMSE receiver" proposed in [3] has been shown to have several advantages over the conventional matched filter based receiver[4]. Its structure is attractive because it can be implemented in a decentralized fashion. Also, it can eliminate multiple access interference as the centralized multiuser detectors while disregarding information on timing, signature, carrier phase and initial synchronization are needed as the conventional receiver.

However, the implementation of the adaptive linear MMSE receiver is originally realized on the fractionally-spaced transversal filters [3] with received signals oversampled by an input sampler in order to moderate the effect of timing phase error. The idea of oversampling received signals is adapted from the concept of the fractionally-spaced equalizer [7-8]. By the realization of oversampling, memory required for receiver coefficients and calculation complexity is increased. Also, additional algorithms such as the tap-leakage algorithm [9] for the stable operation of this receiver may be needed.

In this paper, it is shown that the necessity for oversampling in an adaptive linear MMSE receiver may be dropped by appropriate selection of signature sequences. Moreover, the sensitivity to a timing phase error of signature sequences is also investigated.

II. System Description

We consider a discrete-time, baseband K-user CDMA system as depicted in Fig.1a. We further consider an information symbol of the k-th user $a_k(i) \in \{-1,1\}$ having a period T modulated by an impulse signature sequence assigned to the user ,

$$s_k(t) = \sum_{m=0}^{M-1} c_k(m)\delta(t - mT_c) \tag{1}$$

where T_c is the chip duration referring to the interval during any two adjoining signature symbols, $c_k(m)$ and $c_k(m+1)$. The spreading gain for this signature is thus 1/M. Subsequently, the modulated-impulse signal is filtered by the waveshaping filter $p(t)$ to limit the transmission bandwidth.

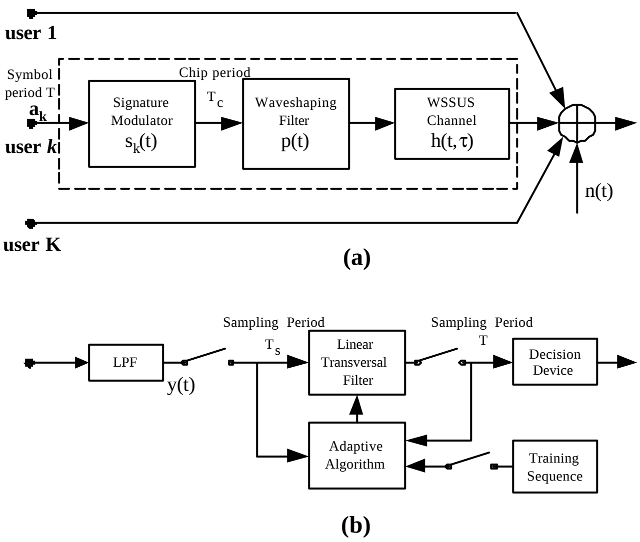

Fig. 1. (a) Transmission of K-user CDMA system.
(b) Adaptive MMSE receiver for the desired k-user

At the receiver-side Fig.1b, the noise rejection filter is first applied to limit noise at the sampler input. The received signal of the K-user asynchronous system at the adaptive MMSE filter may be expressed as

$$y(t) = \sum_{k=1}^{K} \sum_{i=0}^{N-1} a_k(i) u_k(t - iT - \phi_k) + v(t) \tag{2}$$

where $u_k(t) = s_k(t) \otimes p(t) \otimes h(t,\tau)$ refers to the received signature waveform as seen by the adaptive MMSE receiver, ϕ_k is an arbitrary reference to the k-th user; $\phi_k \in [0,T)$, $h(t,\tau)$ is the slow-fading wide-sense-stationary scattering(WS-SUS) channel and $\nu(t)$ is bandlimited additive white Gaussian noise(AWGN) with variance σ^2. The received signal is sampled at rate $1/T_s$ where $T_s = DT_c$ and D is integer. The discrete T_s-spaced sampled output is fed to the adaptive MMSE filter afterwards.

After the adaptive filtering process, a downsampler of the symbol rate T is applied to the adaptive linear filter output, implying a cascaded adaptive linear filter and output sampler serving as a decimating filter.

II. The Adaptive MMSE Receiver

A. The Optimum MMSE Filter Coefficients

Basically, the adaptive MMSE filter can be realized in the form of linear transversal filter. By this, the sampled signal will be observed over a running window of length L; the observed window can be represented in matrix from as follows

$$\mathbf{y}(i) = \mathbf{U}\mathbf{a}(i) + \nu(i) \quad (3)$$

where $\mathbf{U}$ is the matrix of the received sampled signature waveform; $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_K]$, the vector $\mathbf{a}(i) = [\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_K]$ represents transmitted symbols of K users during that window period and $\nu(i)$ is bandlimited AWGN.

By applying the mean-squared error (MSE) criterion [2], the expression of the optimum linear FIR filter coefficients for the desired k-th user is given by [3]:

$$\mathbf{w}_{\text{opt}} = \mathbf{F}^{-1}\mathbf{u}_k \quad (4)$$

where the matrix $\mathbf{F} = \mathbf{U}\mathbf{U}^H + \sigma^2$ is nonsingular [5] and called the *multiple access channel correlation matrix*, and $\mathbf{u}_k = E[a_k \mathbf{y}]$ is the signature vector associated with symbol a_k.

For conventional T_c-spaced sampling or sampling with period $T_s = T_c$, the transfer characteristic of this optimum linear FIR filter was derived in the Appendix A of [3]:

$$\mathcal{W}_k(\omega) = \frac{\mathcal{U}_k^*(\omega)}{\frac{1}{T_c}\sum_{k=1}^{K}\sum_{l=0}^{L-1}\left|\mathcal{U}_k\left(\omega + l\frac{2\pi}{T_c}\right)\right|^2 + \sigma^2} \quad (5)$$

where $\mathcal{U}_k(\omega)$ is the fourier transform of the received signature waveform of the k-th user and the number of filter coefficients is L-1. Note that $\mathcal{U}_k(\omega)$ in the denominator of (5) is the aliased spectrum of sampled signature waveform. However, to consider solely on the spectrum of the interest user; user k, we may write (5) as follows

$$\mathcal{W}_k(\omega) = \frac{\mathcal{U}_k^*(\omega)}{\frac{1}{T_c}\sum_{l=0}^{L-1}\left|\mathcal{U}_k\left(\omega + l\frac{2\pi}{T_c}\right)\right|^2 + \Psi + \sigma^2} \quad (6)$$

where Ψ denotes the *multiple access interference*(MAI).

Moreover, It is assumed that the coefficients spanning LT_c of the linear FIR filter is sufficiently long to collect most of the energy dispersed by the transmission channel and the waveshaping filter.

B. Effects of the Timing Phase Error

Let the k-th user be the user of interest. For simplicity of notation, we may omit the subscript for user k in the sequel. Referring to the model in Fig.1b, the aliased (Nyquist-equivalent) spectrum (noise ignored) of an isolated chip observed by the adaptive MMSE filter is thus

$$\mathcal{U}_{eq,\tau}(\omega) = \sum_l \mathcal{U}\left(\omega + l\frac{2\pi}{T_c}\right)\exp\left(j\left(\omega + l\frac{2\pi}{T_c}\right)\tau\right) \quad (7)$$

where the sampling instants of the input sampler are at $t = iT_c + \tau$. We further assume that if the *excess bandwidth* occupied by the signal beyond the Nyquist frequency caused by the waveshaping filter $p(t)$ is less than 50%, it is then easy to show that only three terms corresponding to $l = 0$, ± 1 will survive. We may thus express the square magnitude of $\mathcal{U}_{eq,\tau}(\omega)$ as

$$|\mathcal{U}_{eq,\tau}(\omega)|^2 = \left|\mathcal{U}_k(\omega)e^{j\omega\tau} + \mathcal{U}_k\left(\omega - \frac{2\pi}{T_c}\right)e^{j\left(\omega - \frac{2\pi}{T_c}\right)\tau}\right|^2 \quad ; |\omega| \le \frac{\pi}{T_c} \quad .(8)$$

There are some observations about $|\mathcal{U}_{eq,\tau}(\omega)|^2$ in (8). First, a null may be created in the rolloff portion $(1-\alpha)\pi/T_c \le |\omega| \le (1+\alpha)\pi/T_c$; that is, the width of this null is determined by the rolloff factor α while its depth depends on τ with the deepest when $\tau = T_c/2$. Note that the null will cause performance degradation because of the noise and MAI enhancement caused by an attempt of the linear FIR filter to compensate for this null.

Secondly, since $\mathcal{U}_{eq,\tau}(\omega)$ is the cascade of the signature modulator, waveshaping filter and transmission channel, the characteristic of each signature sequence may result in the different severity level of the null. This is discussed further in the next session.

To mitigate the effects of noise and MAI enhancement, an oversampling technique may be applied to the input sampler. This will enable the adaptive MMSE filter to compensate for the channel distortion directly, before aliasing and later compensating for any arbitrary timing phase distortion[7-9]. Unfortunately, sampling at higher rate requires more memory for coefficients and complex coefficient calculations are required to keep the same time span of the linear FIR filter; these may be undesired in some circumstances.

C. The Influence of Signature Sequences

Recalling (1), we may write the transfer function of the signature modulator as follows

$$S(\omega) = c_0 + c_1 e^{-j\omega T_c} + \ldots + c_{M-1} e^{-j(M-1)\omega T_c} \quad (9)$$

For the waveshaping filter, we make our consideration practical by applying the raised cosine frequency characteristic:

$$\mathcal{P}(\omega) = \begin{cases} T_c & ;0 \le |\omega| \le (1-\alpha)\frac{\pi}{T_c} \\ \frac{T_c}{2}\left\{1+\cos\left[\frac{T_c}{\alpha}\left(|\omega|-(1-\alpha)\frac{\pi}{T_c}\right)\right]\right\} & \\ & ;(1-\alpha)\frac{\pi}{T_c} \le |\omega| \le (1+\alpha)\frac{\pi}{T_c} \\ 0 & ;|\omega| \ge (1+\alpha)\frac{\pi}{T_c} \end{cases} \quad (10)$$

where α is the rolloff factor. Let $\mathcal{H}(\omega)$ be the transfer function of the cascade of $S(\omega)$ and $\mathcal{P}(\omega)$,

$$\mathcal{H}(\omega) = S(\omega)\,\mathcal{P}(\omega)\,. \quad (11)$$

We show the frequency characteristics $|\mathcal{H}(\omega)|^2$ of several signature sequences when their sampled phases are 0 and $T_c/2$, and the rolloff factor of the waveshaping filter is equal to 0.5 in Fig.2. It is shown that the energy distribution of each sequence depends on the signature sequence. Furthermore, when the received signal is sampled at $\tau = T_c/2$, it is seen that the more energy in the affected region $(1-\alpha)\pi/T_c \le|\omega|\le(1+\alpha)\pi/T_c$, the more energy will be eliminated from this frequency range. For instance, the energy of the sequences in Fig.2a and Fig.2c are concentrated near π/T_c, thus when the received signal is sampled at $\tau = T_c/2$, most of spectrum energy remains. On the other hand, the sequences in Fig.2b and Fig.2d have the energy distribution mainly in $(1-\alpha)\pi/T_c \le|\omega|\le(1+\alpha)\pi/T_c$ band; therefore, when the null is created at this region, the majority of their energy is wiped out.

Since the linear FIR filter will make an inversion of channel response plus MAI and noise, this leads MAI and noise enhancement in that affect region. However, the severity of the enhancement depends on the energy wiped out by the aliasing effects; that is, the signature sequence may be another key role to alleviation enhancement problems without applying oversampling. For instance, the signature sequences in Fig.2b and Fig.2d which are very sensitive to the timing phase error will cause noise enhancement of noise and MAI when applied to the sequences in Fig.2a and Fig.2c will have less enhancement when the timing phase error occurs.

Another point to be noted is that the longer the sequences, the greater the choices (degree of freedom) when selecting the proper signature sequences.

Now the problem of oversampling avoidance is narrowed to how many *good* sequences can be assigned and how much of a trade-off between the cost of complexity for oversampling and the performance degradation for conventional sampling. We will address these through simulation results in the next section.

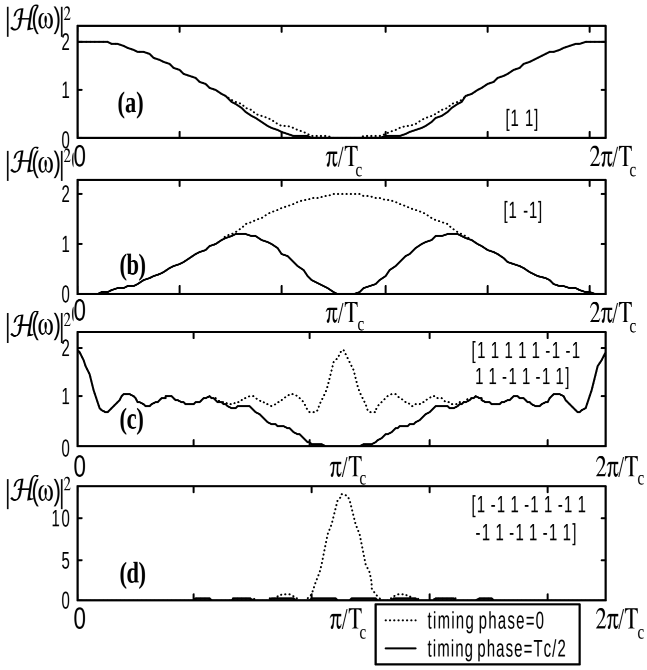

Fig. 2. Plot of $|\mathcal{H}(\omega)|^2$ in the case $\tau = 0$ and $T_c/2$ of the sequences (a)[1 1] (b)[1 -1](c)[1 1 1 1 1 -1 -1 1 1 -1 1 -1 1 1] (d)[1 -1 1 -1 1 -1 1 -1 1 -1 1 -1 1 1]. Note that the rolloff factor α=0.5 for all plots.

III. Numerical Results

CDMA systems with a spreading gain 8 and 12 are considered in Fig.3 and Fig.4, respectively. The performance index, mean-squared error (MSE), is computed at the steady-state in the training process where the least-mean-square (LMS) algorithm is applied. Furthermore, the performance index is observed when the timing phase is at $\tau = 0$, $T_c/4$ and at $\tau = T_c/2$ of T_c-spaced sampling. The timing phase $\tau = 0$ is intentionally chosen to show the perfect case while $\tau = T_c/2$ is for the worst case of sampling. In addition, the MSE of the adaptive MMSE receiver applying oversampling ($T_c/2$-spaced) is also given.

We assume herein that the structure of MAI is ignored and the MAI is assumed to be Gaussian distributed. The signal-to-noise-plus-MAI (SIR) ratio is set to 20 dB. Moreover, the waveshaping filter having a raised cosine spectrum with the rolloff factor $\alpha = 0.5$ is applied. The tails of its impulse response are limited at magnitude of 15 dB below its peak value. Note that although the different choices of the rolloff factor value result in the MSE, by simulation experience results when the rolloff factor in the range 0.1-1.0 are not far different; therefore, the only selected value of $\alpha = 0.5$ is reasonable and sufficient to draw a conclusion.

In Fig.3 and Fig.4, the MSE is plotted in logarithm scale against all possibilities of signature sequences which can be represented from 0 to 255 and 4095 (decimal) for signature sequence of length 8 and 12, respectively. To express the influence of signature sequences explicitly, we have sorted the MSE in ascending order of the T_c-spaced,$\tau = T_c/2$ category so as to present the relation of the number of signature sequences (in worst case of sampling) effecting to each level of the MSE. Then the MSE of the other categories are plotted with respect to the reordered signature sequences. As expected, the MSE of the conventional T_c-spaced sampling (①,②and③) are more sensitive to the timing phase error, while the oversampling categories (④and⑤) show no changes with regard to timing phase errors. Furthermore, the MSE of the oversampling categories (④and⑤) is very close to that of T_c-spaced, no timing phase error category (②).

From the simulation results, the problems arising from the previous section about the trade-off between the conventional sampling and oversampling, and how many good sequences existing can be answered. We note that the worst category; T_c-spaced, $\tau = T_c/2$, will be used for the following comparison. In Fig.3, there are approximately 42(16%) signature sequences can be used for T_c-spaced sampling indicating the MSE degradation within 1dB of those oversampling whereas we can apply 140(55%) signature sequences for T_c-spaced sampling yielding the MSE degradation worse within 3dB of those oversampling. Similarly, in Fig.4 there are about 625(15%) and 2300(56%) signature sequences can be used for T_c-spaced sampling giving the MSE degradation within 1dB and 2dB, respectively compared to oversampling data. These results are quite similar for both sequences of length 8 and 12. We also note that the sequences showing high sensitiveness to the timing phase error have their energy distribution mainly in $(1-\alpha)\pi/T_c \leq|\omega|\leq(1+\alpha)\pi/T_c$ region. Not surprisingly, the two signature sequences giving the largest MSE for T_c-spaced sampling, when $\tau = T_c/2$ in Fig.3 and Fig.4 are the sequences alternating between 1 and -1; [1 -1 1 -1 ...] and [-1 1 -1 1 ...], of length 8 and 12, respectively.

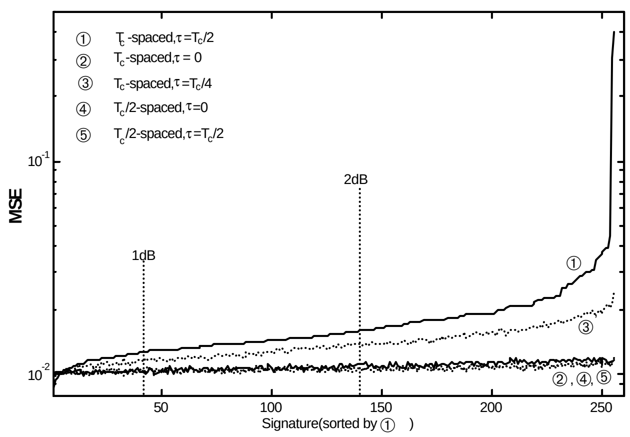

Fig. 3. Plot of MSE versus (sorted) signature sequences of length 8 and SNIR=20dB

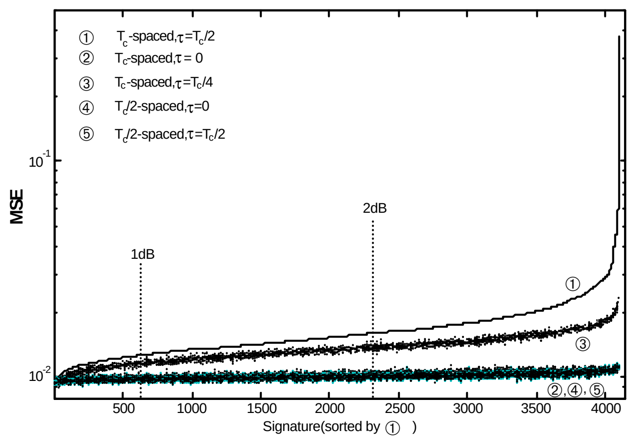

Fig. 4. Plot of MSE versus (sorted) signature sequences of length 12 and SNIR=20dB

Since the simulation results show the degree of timing phase error sensitiveness of each sequence where some sequences are acutely sensitive to the timing phase error; therefore, the avoidance of these sequences for conventional T_c-spaced sampling can prevent the system from vicious degradation of performance.

As described above, the Fig.3 and Fig.4 explicitly show the trade-off between complexity increased from implementing the oversampling and the performance degradation from applying the conventional T_c-spaced sampling.

V. CONCLUSIONS

The avoidance of oversampling for adaptive linear MMSE receivers for CDMA systems (Gaussian-distributed MAI assumed) is addressed. The simulation results have shown that by applying the conventional T_c-spaced sampling, each signature sequence responds differently to the timing phase error with different value of the mean-squared error (MSE) and some sequences are highly sensitive to the timing phase error. By selection of the signature sequences that are less responsive to the timing phase error with an acceptable level of performance degradation, we can have a reasonable trade-off between the cost of complexity of oversampling implementation and the performance degradation of conventional T_c-spaced sampling. In other words, each signature sequence is different in sensitiveness to the timing phase error when chip-rate sampling is applied. The less sensitive sequences yield the MSE close to the oversampling case or chip-rate sampling with perfect timing phase.

Although the results of the degradation level shown in Fig.3 and 4 are the special case when the rolloff factor value $\alpha = 0.5$ and varying the rolloff factor α may change the MSE, the same conclusion can still be drawn as above.

In addition, we have observed that the desired signature sequences should have spectrum energy concentrated as less as possible in the rolloff region $(1-\alpha)\pi/T_c \leq|\omega|\leq(1+\alpha)\pi/T_c$ where α is the rolloff factor. This can be a criteria for the signature sequence design for CDMA systems applying adaptive linear MMSE receivers where the T_c-spaced sampling is employed.

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