New Rate-1/2 Transparent Convolutional Codes over Ring Z_8 for 8-PSK Signals

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ABSTRACT

New rate-1/2 transparent systematic linear convolutional codes over ring Z_8 for 8-PSK signals having 1.5 bit/sec/Hz of bandwidth efficiency for bandlimited additive white Gaussian noise channels are reported. These codes give good coding gains, low decoding complexity, and they are suitable for use in many practical bandlimited channels with phase offset.

1. INTRODUCTION

In combined convolutional channel coding and modulation design, 2-dimensional and multi-dimensional coded modulation techniques were proposed for sending integer and non-integer number of information bits per modulation interval, respectively [1, 2]. In an earlier paper, the concept of using 2-dimensional coded modulation technique, with relatively low decoding complexity, to send non-integer number of information bits per modulation interval was presented [3]. New rate-1/2 non-transparent feedforward and feedback systematic linear convolutional codes over ring Z_8 for 8-PSK signals having 1.5 bit/sec/Hz of bandwidth efficiency for bandlimited additive white Gaussian noise channels were reported.

In a coded phase modulation system with phase offset problems, codes should be made transparent to signal rotations. Transparent codes can be utilised together with a differential coding process in order to eliminate errors which can be caused by the phase offset. In this paper, we report new rate-1/2 transparent feedforward and feedback systematic linear convolutional codes over ring Z_8 of encoder memory order [4] 1, 2, .., 4 with 8-PSK signals having 1.5 bit/sec/Hz of bandwidth efficiency for bandlimited AWGN channels. These codes are optimum in the sense that they have the maximum asymptotic coding gains (ACG) and minimum average multiplicity of squared free Euclidean distance error events for the given rate and encoder memory order. The asymptotic coding gain of the rate-1/2 transparent codes over ring Z_8 with 8-PSK signals are compared with the rate-1/2 non-transparent codes over ring Z_8 found in [3], and the uncoded coherent PSK signals.

2. ENCODER MODEL AND ASSUMPTIONS

A rate-$(n$-$1)/n$ systematic linear convolutional code of encoder memory m over ring Z_q can be represented by the generator polynomial matrix

$$\boldsymbol{G}(D) = \begin{bmatrix} 1 & 0 & \cdots & 0 & G_1(D)/F(D) \\ 0 & 1 & \cdots & 0 & G_2(D)/F(D) \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 1 & G_{n-1}(D)/F(D) \end{bmatrix}$$

Here, D is the unit-delay operator, $G_i(D) = g_{i,0} + g_{i,1}D + ... + g_{i,m}D^m$ for $1 \le i \le (n - 1)$ and $F(D) = 1 + f_1D + ... + f_mD^m$. The coefficients of the polynomials belong to the set $Z_q = \{0, 1, ..., q - 1\}$. Z_q is defined as a ring of integers modulo-q. Fig. 1 shows the encoder circuit of a rate-$(n$-$1)/n$ systematic linear convolutional code over ring Z_q. The components of the information vector $\boldsymbol{X}_l = [x_l^{(1)}\ x_l^{(2)} ... x_l^{(n-1)}]$ and the encoded vector $\boldsymbol{Y}_l = [y_l^{(1)}\ y_l^{(2)} ... y_l^{(n)}]$, for $l \ge 0$, are symbols over ring Z_q. It is assumed that the element $y_l^{(j)}$, $1 \le j \le n$, is mapped uniquely onto a q-ary PSK signal constellation diagram using natural-binary mapping. All the signal points s_0, s_1, ..., s_{q-1} are equally spaced and lie on the unit circle in the signal constellation diagram. The average signal symbol energy E_s is unity.

Theorem. A linear convolutional code over ring Z_q is transparent (invariant) to $2\pi/q$ phase variation if and only if the all-one sequence is a codeword [5].

Proof. The modulo-Z_q addition of any codeword $\boldsymbol{c}$ of the code C and the all-one codeword **1** followed by a multiplication of a scalar r, $0 \leq r < Z_q$, is also a codeword $\boldsymbol{c}'$ in C. The codeword $\boldsymbol{c}'$ is a rotated verison of $\boldsymbol{c}$. □

The asymptotic coding gain of this coded modulation scheme with unquantised maximum-likelihood decoding for AWGN channels over the corresponding uncoded modulation having the same average signal power is $10 \log_{10} [d^2_{fed}/d^2_{uncoded}]$, where d^2_{fed} is the minimum squared free Euclidean distance between two possible signal sequences in the coded case and $d^2_{uncoded}$ is the minimum squared Euclidean distance between two nearest signal points in the uncoded case.

3. CODE SEARCH RESULTS

Tables 1 and 2 give the encoder memory order m, the number of trellis states, the generator polynomial coefficients, the minimum squared free Euclidean distance d^2_{fed}, the number of nearest neigbours N_{fed}, and the asymptotic coding gains, ACG (dB), of the codes. Column 3 of Tables 1 and 2 gives the coefficients of $G_1(D)$ and $G_1(D)/F(D)$, respectively. For example, rate-1/2 feedforward and feedback systematic convolutional codes of $G_1(D) = 2 + 3D + 4D^2$ and $G_1(D)/F(D) = (3 + 2D^2)/(1 + 2D + 2D^2)$ are expressed as 2 3 4 and 3 0 2/1 2 2, respectively. Essentially, an exhaustive code search procedure is used to find good transparent systematic feedforward and feedback linear convolutional codes.

Tables 3 and 4 compare the parameters of the rate-1/2 transparent feedforward and feedback systematic linear convolutional codes over Z_8 with the corresponding rate-1/2 non-transparent codes over Z_8 found in [3]. As expected, for the same number of trellis state, the rate-1/2 transparent code over ring Z_8 has d^2_{fed} smaller than or equal to the corresponding rate-1/2 non-transparent code over ring Z_8 with 8-PSK signals. The rate-1/2 transparent codes over ring Z_8, presented here, achieve very useful coding gains over the uncoded modulation schemes.

4. CONCLUSION

In this paper, new rate-1/2 transparent feedforward and feedback systematic linear convolutional codes over ring Z_8 for 8-PSK signals giving 1.5 data bits per modulation signal symbol on bandlimited additive white Gaussian noise channels were reported. These codes, with relatively low decoding complexity (say less than 64 states), give good coding gains and they are suitable for use in many practical bandlimited channels with phase offset.

5. REFERENCES

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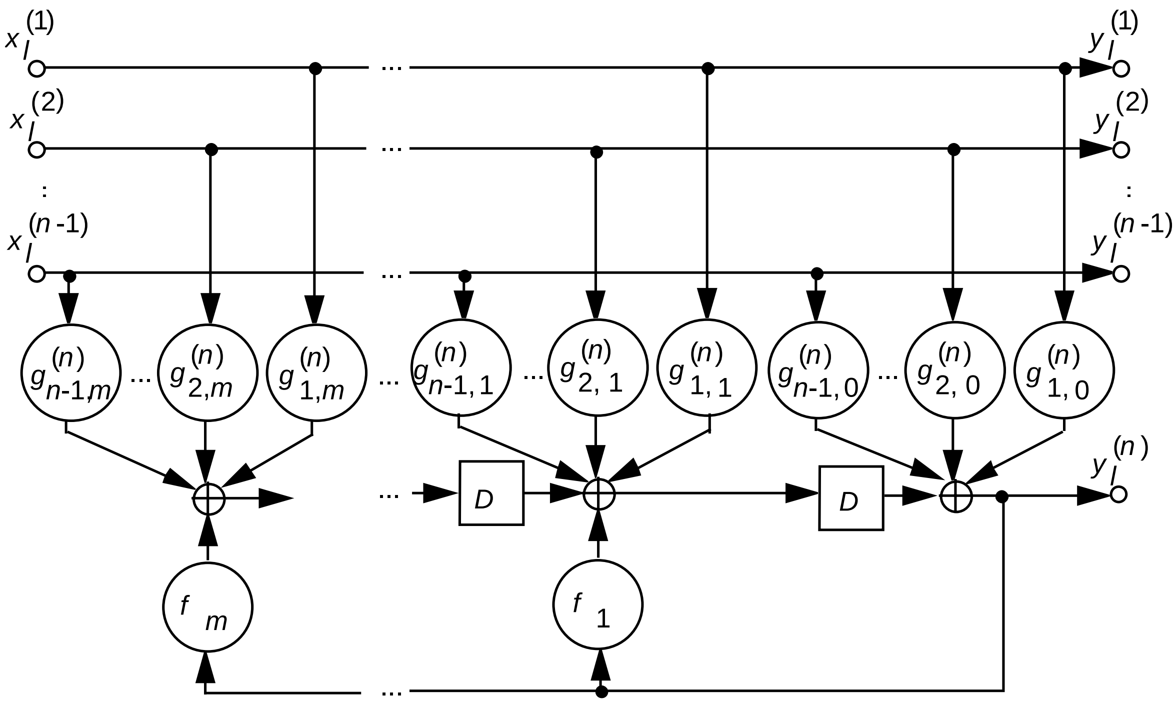

Figure 1 (n, n-1, m) systematic linear convolutional encoder with feedback.

m	States	Generator polynomial coefficients	d^2_{fed}	N_{fed}	ACG_{BPSK} (dB)	$ACG_{\text{4-PSK}}$ (dB)
1	2	5 4	4.00	2	0.00	3.01
1	4	3 6	6.00	4	1.76	4.77
2	8	2 3 4	7.17	4	2.54	5.55
2	16	5 2 2	8.00	5	3.01	6.02
4	32	2 5 6 0 4	8.00	1	3.01	6.02
3	64	3 5 7 2	10.00	4	3.98	6.99

Table 1 PARAMETERS OF RATE-1/2 TRANSPARENT FEEDFORWARD CONVOLUTIONAL CODES OVER RING Z_8

m	States	Generator polynomial coefficients	d^2_{fed}	N_{fed}	ACG_{BPSK} (dB)	$ACG_{\text{4-PSK}}$ (dB)
1	2	5 /1 4	4.00	2	0.00	3.01
1	4	3 /1 6	6.00	4	1.76	4.77
2	8	3 0 2 /1 2 2	8.00	5	3.01	6.02
2	16	3 1 1 /1 1 3	8.00	1	3.01	6.02
2	32	3 5 /1 4 2	9.17	2	3.60	6.61
3	64	2 4 3 /1 7 1	11.17	8	4.46	7.47

Table 2 PARAMETERS OF RATE-1/2 TRANSPARENT FEEDBACK CONVOLUTIONAL CODES OVER RING Z_8

States	Rate-1/2 transparent feedforward convolutional codes over ring Z_8			Rate-1/2 non-transparent feedforward convolutional codes over ring Z_8		
	d^2_{fed}	N_{fed}	ACG_{BPSK} (dB)	d^2_{fed}	N_{fed}	ACG_{BPSK} (dB)
2	4.00	2	0.00	4.00	1	0.00
4	6.00	4	1.76	6.00	4	1.76
8	7.17	4	2.54	7.17	4	2.54
16	8.00	5	3.01	8.00	5	3.01
32	8.00	1	3.01	8.59	2	3.32
64	10.00	4	3.98	10.00	4	3.98

Table 3 COMPARISONS BETWEEN BEST RATE-1/2 TRANSPARENT AND NON-TRANSPARENT FEEDFORWARD CONVOLUTIONAL CODES OVER RING Z_8

States	Rate-1/2 transparent feedback convolutional codes over ring Z_8			Rate-1/2 non-transparent feedback convolutional codes over ring Z_8		
	d^2_{fed}	N_{fed}	ACG_{BPSK} (dB)	d^2_{fed}	N_{fed}	ACG_{BPSK} (dB)
2	4.00	2	0.00	4.00	1	0.0
4	6.00	4	1.76	8.00	13	3.01
8	8.00	5	3.01	8.00	5	3.01
16	8.00	1	3.01	9.17	4	3.60
32	9.17	2	3.60	10.34	10	4.13
64	11.17	8	4.46	11.51	8	4.59

Table 4 COMPARISONS BETWEEN BEST RATE-1/2 TRANSPARENT AND NON-TRANSPARENT FEEDBACK CONVOLUTIONAL CODES OVER RING Z_8