Decision Feedback Equalizer Based Multiuser Detection in DS-CDMA Systems

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Abstract—**In this paper, we investigate the architecture and performance of a decision feedback equalizer (DFE) based multiuser receiver for asynchronous direct sequence code division multiple access (DS-CDMA) systems. The DFE consists of a feedforward filter (FFF) and a feedback filter (FBF) to combat the intersymbol interference (ISI) and multiple access interference (MAI). We use an adaptive, centralized decision feedback scheme where, in addition to the feed forward estimates, the previous and current decisions of the interfering users' data are taken into account. We evaluate the performance of the DFE based receiver on both AWGN as well as Rayleigh fading channels in a *near-far* scenario. The proposed DFE receiver structure is shown to exhibit good near-far resistance and offer significant performance advantage over linear adaptive receivers which use only feedforward filters.**

I. INTRODUCTION

Direct sequence code division multiple access (DS-CDMA) has become a popular alternative to traditional multiple access techniques like FDMA and TDMA [1]. In DS-CDMA, several asynchronous users simultaneously transmit their data by modulating the data with user specific signature waveforms. Demodulation of DS-CDMA signals is conventionally achieved by a bank of filters, each matched to the signature waveform of a user. The conventional matched filter receiver (CMFR) suffers from the *near-far problem*, whereby a weak signal from a distant user is overwhelmed by a stronger signal from a nearer interferer which causes significant performance degradation. When a CMFR is used, the approach to dealing with the near-far problem has been to use transmitter power control [1],[2]. On the other hand, *multiuser detection* (MUD) receivers can provide significantly better near-far resistance (i.e., interference rejection capability) than CMFRs [3]. While the CMFR treats MAI as merely noise and detect only the data of the user-of-interest, MUD receivers jointly detect all users' data. The improved interference rejection capability offered by the MUD receivers can directly result in increased capacity in DS-CDMA systems.

The optimum multiuser detector, proposed by Verdu [4], has a computational complexity that grows exponentially with the number of active users in the system. In addition, the optimum detector requires the knowledge of the spreading codes, signal amplitudes, and propagation delays of all the users. Consequently, a lot of recent research has been focussed on sub-optimum, low-complexity multiuser detectors [5]. The adoption of MUD receivers as an option in the 3G cellular standards [6] to increase system capacity has further enhanced the interest in sub-optimum, low-complexity MUD receivers.

One sub-optimum multiuser receiver on which extensive research has been done is the linear minimum mean square error (MMSE) receiver which has a linear complexity, and adaptive techniques (e.g., LMS, RLS algorithms) to approach the MMSE solution are suitable for implementation [7]. The application of nonlinear receivers for interference rejection in DS-CDMA systems has been an active area of recent research. In this paper, we focus on the architecture and performance of a decision feedback equalizer (DFE) for multiuser detection in DS-CDMA systems. The DFE has a nonlinear structure which consists of two parts: a feedforward filter (FFF) which operates on the outputs of the chip matched filter, and a feedback filter (FBF) which operates on the past decisions of the desired user's data as well as those of the interfering users' data. Although only past data decisions are fed back in conventional DFE receivers, in this paper we consider feeding back the current data decisions of the stronger interfering users as well in estimating the data of the desired user. We evaluate the performance of the proposed DFE based receiver on both AWGN as well as Rayleigh fading channels in a *near-far* scenario, and compare it with that of linear adaptive receivers which have only feedforward filters. The proposed receiver structure is shown to exhibit good near-far resistance and offer significant performance advantage over linear adaptive receivers.

The rest of this paper is organized as follows. In Section II, we present the DS-CDMA system model considered. In Section III, we present the DFE receiver structure. Per-

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formance results and discussions are presented in Section IV. Conclusions are presented in Section V.

II. SYSTEM MODEL

We consider an asynchronous DS-CDMA system with K active users in the system. Binary phase shift keying (BPSK) with bit duration T_b is assumed. The signature waveforms are assumed to be periodic with period $T_b = NT_c$, where T_c is one chip duration and N is an integer. N is the processing gain of the system. The signature waveform of the k^{th} user is given by

$$b_k(t) = \sum_{n=0}^{N-1} c_k(n)\psi(t - nT_c), \quad 0 \le t \le T_b, \qquad (1)$$

where $c_k(n) \in \{-1, +1\}$ is the n^{th} chip of the k^{th} user's spreading sequence, $\psi(t)$ is the chip waveform of duration T_c and unit energy. The baseband signal of the k^{th} user, $s_k(t)$, is formed by modulating the data stream $d_k(i) \in \{-1, +1\}$ with the signature waveform, i.e.,

$$s_k(t) = \sum_{i=-\infty}^{\infty} d_k(i) b_k(t - iT_b)\,. \qquad (2)$$

The baseband signal $s_k(t)$ is multiplied with the carrier $\sqrt{2P_k}\cos(\omega_c t + \theta_k^{'})$, and transmitted. P_k is the transmit power of the k^{th} user, $\theta_k^{'}$ is the random carrier phase of the k^{th} user, which is uniformly distributed in $[0, 2\pi)$, and ω_c is the carrier frequency. The received signal due to the k^{th} user is of the form

$$r_k(t) = \sqrt{2P_k}\,\alpha_k(t)\, s_k(t - \tau_k)\cos(\omega_c t + \theta_k)\,, \qquad (3)$$

where $\alpha_k(t)$ represents the envelope of the fading process for the k^{th} user, $\tau_k \in [0, T_b)$ is the propagation delay of the k^{th} user and $\theta_k = \theta_k^{'} - \omega_c\tau_k + \phi_k(t)$ with $\phi_k(t)$ being the random phase associated with the fading process. The composite received signal, $r(t)$, from K users is then given by

$$r(t) = \sum_{k=1}^{K} r_k(t) + n(t)\,, \qquad (4)$$

where $n(t)$ represents additive white Gaussian noise with zero mean and two sided power spectral density $N_o/2$ W/Hz.

III. DFE RECEIVER STRUCTURE

The DFE receiver block diagram is shown in Fig.1. The main components are a bank of feedforward filters (FFF) and a centralized feedback filter (FBF). The input to the FFF's are sampled outputs of chip matched filters (CMF) as shown. To have the detected symbols clocked out in synchronism, the input to each user's CMF is a delayed version of the received signal $r(t)$, so that $r(t)$ is delayed by $jT_b - \tau_k$ for the k^{th} user's CMF. τ_k is written as $\tau_k = p_kT_c + \delta_k$, where p_k is an integer and $\delta_k \in [0, T_c)$, and j is the smallest integer which results in a positive processing delay. Assuming $\tau_1 = \theta_1 = 0$ for the first user, the input vector for the m^{th} input bit to the first user's FFF is given by,

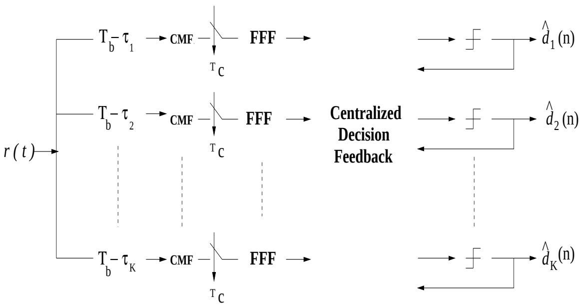

Fig. 1. Block diagram of the Decision Feedback Equalizer

$$\mathbf{r}_1(m) = \sum_{k=1}^{K} \alpha_k(m)\, d_k(m)\, \tilde{\mathbf{I}}_k(m)\sqrt{P_k}\cos\theta_k\; + \mathbf{n}(m)\,, \qquad (5)$$

where $\alpha_k(m) = 1$ for an AWGN channel and is an i.i.d random variable with a Rayleigh distribution for a flat fading channel, and

$$\tilde{\mathbf{I}}_k(m) = \begin{cases} \frac{\delta_k}{T_c}\mathbf{c_k}^{p_k+1} + \left(1 - \frac{\delta_k}{T_c}\right)\mathbf{c_k}^{p_k}; & \text{for } d_k(m-1) = d_k(m), \\ \frac{\delta_k}{T_c}\hat{\mathbf{c}}_{\mathbf{k}}^{p_k+1} + \left(1 - \frac{\delta_k}{T_c}\right)\hat{\mathbf{c}}_{\mathbf{k}}^{p_k}; & \text{for } d_k(m-1) = -d_k(m) \end{cases} \qquad (6)$$

The vectors $\mathbf{c_k}^m$ and $\hat{\mathbf{c}}_{\mathbf{k}}^m$ are given by,

$$\begin{aligned} \mathbf{c_k}^m &= \left[c_k(N-m), \cdots, c_k(N-1), c_k(0), \cdots, c_k(N-m-1)\right]^T, \\ \hat{\mathbf{c}}_{\mathbf{k}}^m &= \left[-c_k(N-m), \cdots, -c_k(N-1), c_k(0), \cdots, c_k(N-m-1)\right]^T \end{aligned}$$

The sequence $c_k(n)$ is found as

$$c_k(n) = \frac{1}{T_c}\int_{nT_c}^{(n+1)T_c} b_k(t)\,dt\,, n = 0, 1, 2, ...N-1 \qquad (7)$$

The noise vector, $\mathbf{n}(m)$, is a Gaussian vector with zero mean, and

$$E[\mathbf{n}(p)\mathbf{n}^T(q)] = \begin{cases} \sigma^2\mathbf{I_N} & \text{if } p = q, \\ \mathbf{0} & \text{if } p \ne q\,, \end{cases} \qquad (8)$$

where $\mathbf{I_N}$ is $N \times N$ identity matrix and the noise variance $\sigma^2 = N\frac{N_o}{2\,E_{b,1}}$. $E_{b,1} = P_1T_b$ is the received energy per bit of the first user.

Figure 2 shows a detailed block diagram of the FFFs and the FBFs. The FFF has N_f taps and the FBF has N_b taps. The input to the FFFs are the outputs from the chip matched filters and the input to the FBFs are the past and current data decisions of the other users. We now define the following vectors:

$$\begin{aligned}
\mathbf{r}_k &= \left[r_k(nT), r_k(nT+Tc), \cdots, r_k(nT+(N_f-1)T_c)\right]^T \\
\mathbf{z}_k &= \left[d_k(n), d_k(n-1), \cdots, d_k(n-(N_b-1))\right]^T \\
\mathbf{w}_k &= \left[w_k(1), w_k(2), \cdots, w_k(N_f)\right]^T \\
\mathbf{f}_{kl} &= \left[f_{kl}(0), f_{kl}(2), \cdots, f_{kl}(N_b-1)\right]^T \ , \qquad (9)
\end{aligned}$$

where $\mathbf{r}_k$ is the input vector to the k^{th} user's FFF, $\mathbf{z}_k$ is the input vector to the k^{th} user's FBF, $\mathbf{w}_k$ is the set of weighting coefficients for the k^{th} user's FFF and $\mathbf{f}_{kl}$ is the set of weighting coefficients from the l^{th} user's FBF to the k^{th} user's summing block.

With the above formulation, the feedforward estimate, which is the output of the FFF for the k^{th} user, is given as $\mathbf{w}_k^T\mathbf{r}_k$. The feedback estimate for the k^{th} user is given as $\mathbf{f}_k^T\mathbf{z}$, where

$$\begin{aligned}
\mathbf{f}_k &\triangleq \left[\mathbf{f}_{k1}^T, \mathbf{f}_{k2}^T, \cdots, \mathbf{f}_{kK}^T\right]^T \ , \\
\mathbf{z} &\triangleq \left[\mathbf{z}_1^T, \mathbf{z}_2^T, \cdots, \mathbf{z}_K^T\right]^T \ . \qquad (10)
\end{aligned}$$

Note that $\mathbf{f}_{ki}$ is an N_b element column vector and $\mathbf{f}_k$ is a KN_b element column vector formed by cascading the $\mathbf{f}_{ki}$ vectors, $i = 1, \cdots, K$. The soft estimate of the n^{th} data bit of k^{th} user is then the difference of the forward estimate and the backward estimate, i.e., $\hat{d}_k(n) = \mathbf{w}_k^T\mathbf{r}_k - \mathbf{f}_k^T\mathbf{z}$.

We optimize the coefficients for each user so as to minimize that user's mean square error (MSE). On flat fading channels, the MSE of the k^{th} user is given as

$$\begin{aligned}
MSE_k &= E\left[(\hat{\alpha_k}(n)\, d_k(n) - \hat{d}_k(n))^2\right] \\
&= E\left[(\hat{\alpha_k}(n)\, d_k(n) - \mathbf{w}_k^T\mathbf{r}_k + \mathbf{f}_k^T\mathbf{z})^2\right] \ , \qquad (11)
\end{aligned}$$

where $\hat{\alpha_k}(n)$ is the estimate of $\alpha_k(n)$. Here, we assume that perfect estimates of the fades are available at the receiver. The optimum weighting coefficient vectors can be approached through adaptive means so that the MSE given in Eqn.(11) is minimized. We employ the normalized least mean square error (NLMS) algorithm for this adaptation [9]. Note that the feedforward weighting coefficients and the feedback weighting coefficients can be adapted independently. It is also possible to adapt the feedforward and feedback coefficients together using a single NLMS algorithm. Our approach here is to adapt the feedforward and feedback coefficients using a single NLMS algorithm.

Let $\mathbf{x}_k \triangleq \left[\mathbf{r}_k^T, \mathbf{z}^T\right]^T$ and $\mathbf{g}_k \triangleq \left[\mathbf{w}_k^T, -\mathbf{f}_k^T\right]^T$. Then, $\hat{d}_k(n) = \mathbf{g}_k^T\mathbf{x}_k$. Note that $\mathbf{x}_k$ and $\mathbf{g}_k$ are column vectors of size $N_f + KN_b$. The NLMS updating algorithm is then given by

$$\begin{aligned}
\mathbf{g}_k(n+1) &= \mathbf{g}_k(n) + \frac{\tilde{\mu}}{\mathbf{x}_k^T(n)\mathbf{x}_k(n)}\mathbf{x}_k(n) \\
&\quad \cdot \Big(\hat{\alpha}_k(n)\, d_k(n) - \hat{d}_k(n)\Big) \ , \qquad (12)
\end{aligned}$$

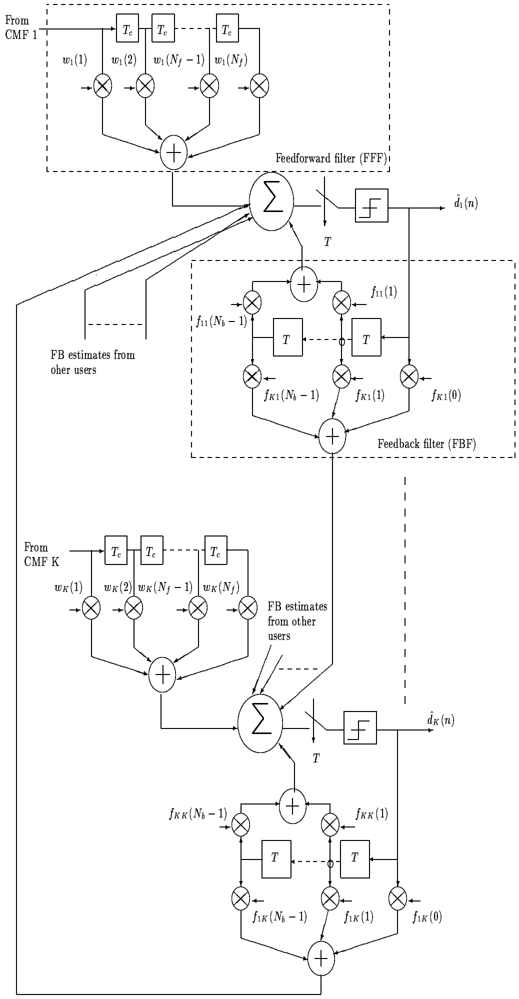

Fig. 2. DFE multiuser receiver structure

where $\tilde{\mu}$ is a constant step size. The above DFE multiuser receiver requires N_f multiplications and $N_f - 1$ additions for every feedforward estimate, and every feedback estimate requires KN_b multiplications and $K(N_b-1)+K-1$ additions. It is noted that the linear adaptive receivers process only the forward estimates (and do not have feedback estimates) and each forward estimate requires N_f multiplications and $N_f - 1$ additions. Thus, there is an additional complexity in the DFE receiver due to the feedback estimates. As we will see in the next Section, using just two feedback taps (i.e., $N_b = 2$) provides significant performance advantage, and the resulting additional complexity of $2K$ multiplications and $2K - 1$ additions is rather small in practical systems which typically have only tens of simultaneous active users in a cell.

IV. RESULTS AND DISCUSSION

In this section, we evaluate the performance of the DFE multiuser detector and compare it with that of CMFR and linear adaptive receiver. We compare the convergence characteristics, bit error rate (BER) performance and capacity characteristics. Simulations were carried out using spreading codes of period $N = 15$. We set the near-far ratio (NFR)[1] of all interfering users to be the same, i.e., all interfering users are received at the same power level ($P_2 = P_3 = \cdots = P_K$). The number of feedforward taps, N_f, was set to N (i.e., the number of chips per bit), and the feedback taps from each user, N_b, was set to 2. We estimate the stronger users first and use these decisions to help estimate the data of weaker users. In all simulations, we assume that the past decisions of all the users' data bits are correct.

Figures 3 and 4 show a comparison of the convergence characteristics of the DFE based receiver and the linear adaptive receiver with $K = 7$, E_b/N_o = 10 dB and NFR = 20 dB on AWGN and flat fading channels, respectively. The following observations can be made from these plots. Firstly, we note that the multiuser receiver with decision feedback converges faster than the adaptive linear receiver. While the linear adaptive receiver takes about 500 bits to converge, the DFE based receiver converges within 250 bits. Secondly, the DFE multiuser receiver results in much smaller MSE values than the linear adaptive receiver. For example, the MSE of the DFE receiver is about 4 dB lesser than that of the linear adaptive receiver. Figure 4 also highlights better convergence properties on a fading channel.

Figures 5 and 6 show the plots of BER versus E_b/N_o with NFR = 20 dB on AWGN and fading channels, respectively. One thousand bits are used to train the adaptive receivers. It is observed that the CMFR performs the worst because of its sensitivity to near-far effect. The linear adaptive receiver performs much better than the CMFR, but performs poorly as compared to the DFE multiuser receiver. The better performance of the DFE based receiver is due to the smaller MSE it produces as compared to the linear adaptive receiver. Figures 7 and 8 show the plots of BER versus NFR, with E_b/N_o = 10 dB on AWGN and fading channels, respectively. As the NFR increases, the CMFR performs poorly. On the other hand, the adaptive receivers perform increasingly better relative to the CMFR for increasing NFR values. The DFE multiuser receiver is seen to have a better near-far resistance when compared to the linear adaptive receiver. Figures 9 and 10 show the capacity characterisics for AWGN and fading channels respectively. The E_b/N_o was set to 10 dB and the NFR to 20 dB. We observe that for a small number of users, the DFE multiuser receiver and the linear adaptive perform almost similarly. However, as the number of users in the system increases, the DFE performs better in both AWGN as well as fading scenarios. The DFE based receiver can therefore be used to increase capacity of DS-CDMA networks, including 3G systems.

V. CONCLUSIONS

We presented the architecture and performance of a DFE based multiuser receiver for asynchronous DS-CDMA systems. We used an adaptive, centralized decision feedback scheme where the previous and current decisions of the interfering users' data are taken into account. We evaluated the performance of the DFE based receiver on both AWGN as well as Rayleigh fading channels in a *near-far* scenario. The proposed DFE receiver structure was shown to exhibit good near-far resistance and offer significant performance advantage over linear adaptive receivers. The performance of the proposed DFE multiuser receiver on a frequency selective fading channel can be investigated further. The effect of incorrect past data decisions on the DFE performance can be investigated further.

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[1] NFR is defined as the ratio of the received power of interfering user to the received power of the desired user, i.e., NFR = $\frac{P_i}{P_1}$, $i \neq 1$, where user 1 is taken as the desired user. When $P_i = P_1$, NFR = 0 dB.

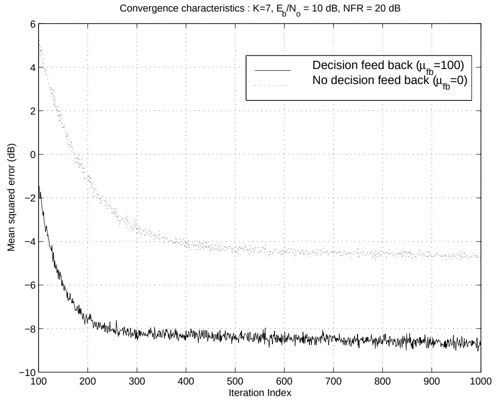

Fig. 3. Convergence comparison of DFE based receiver and linear adaptive receiver: AWGN channel

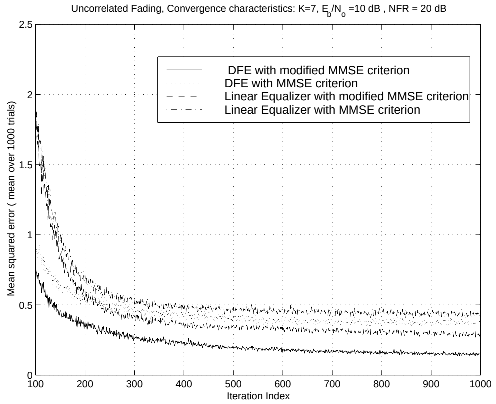

Fig. 4. Convergence comparison of DFE based receiver and linear adaptive receiver: Fading channel

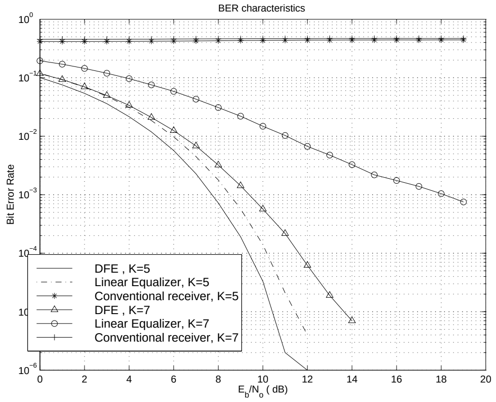

Fig. 5. BER $versus$ Eb/N_o comparison of receivers: AWGN channel

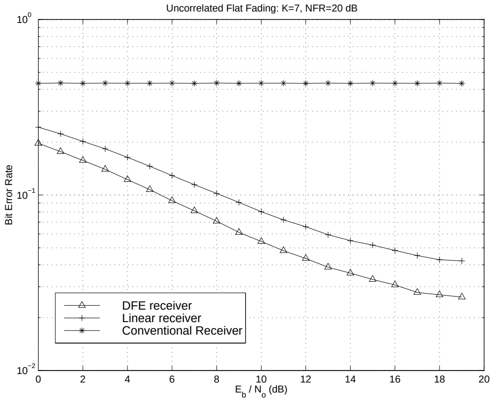

Fig. 6. BER $versus$ E_b/N_o comparison of receivers: Fading Channel

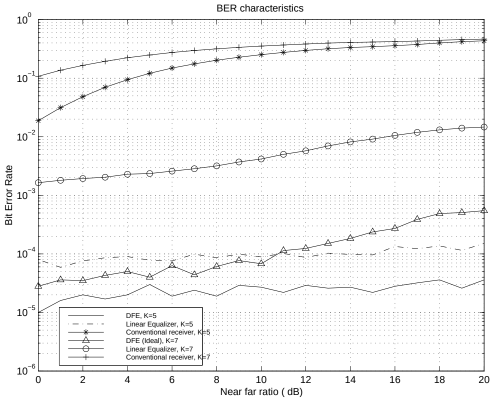

Fig. 7. BER $versus$ NFR comaprison of receivers: AWGN channel

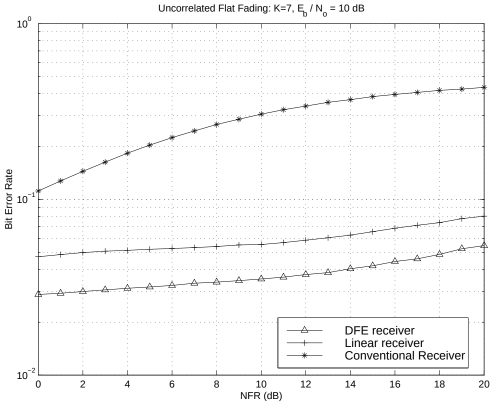

Fig. 8. BER $versus$ NFR comparison of receivers: Fading channel

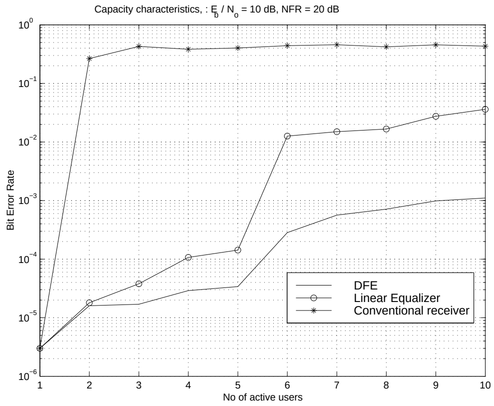

Fig. 9. Capacity comaprison of receivers: AWGN channel

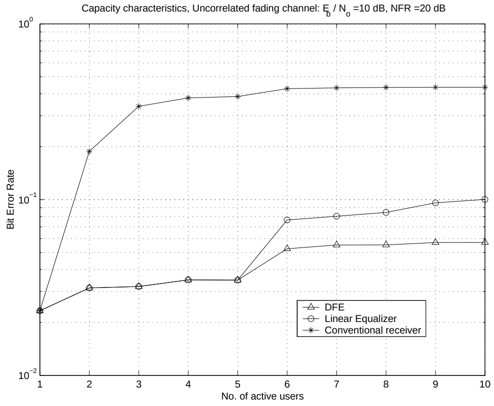

Fig. 10. Capacity comparison of receivers: Fading Channel