An approximate matrix inversion for multiuser projection based detector in CDMA systems

Jinho Choi
Dept. of Electrical and Electronic Engineering
Adelaide University
North Terrace, Adelaide, SA 5005
Email: jinho.choi@adelaide.edu.au

Abstract

In order to reduce the computational complexity, an approximate matrix inversion has been considered for the construction of a multiuser projection based detector in code division multiple access (CDMA) systems. It has been shown that the approximation is improved when the users whose spreading codes are highly correlated are included in constrained user set. From this, the performance is not significantly degraded although the approximation has been utilized.

1 Introduction

The multiuser detection has been investigated to improve the performance in code division multiple access (CDMA) systems [4]. It is known that the optimum multiuser detector is prohibitively complex. Hence, suboptimal multiuser detectors have been proposed. For example, the decorrelating detector has been proposed in [1]. The computational complexity of the decorrelating detector is much lower than that of the optimal detector as the number of users gets larger. To implement the decorrelating detector, a matrix inversion is required for each symbol interval. In some applications, the required computational complexity for the decorrelating detector can be still high. Hence, in [2], the use of an approximate matrix inversion has been proposed to avoid matrix inversion in the decorrelating detector.

In [3], a suboptimal multiuser detector, called projection receiver, has been proposed. It is shown that it performs better than the decorrelating detector. In order to implement the projection detector (in this paper, the projection receiver is referred to as the projection detector), matrix inversion is required to find projection matrix. Clearly, it has high computational burden to implement. In this paper, we consider the use of approximate matrix inversion for the projection detector to reduce the computational complexity. Due to approximation, however, the performance shall be degraded. In order to reduce this performance degradation, we propose an approach: the users whose spreading codes are highly correlated are included into the constrained user set. We confirm that the performance has been improved through computer simulations.

2 Multiuser projection detector

From [4], a well-known model for K-user synchronous CDMA channel output after matched filtering to the common chip waveform provides the linear relationship as

$$\mathbf{y} = \mathbf{AWd} + \mathbf{n}, \tag{1}$$

where $\mathbf{y}$ is an $N \times 1$ output vector, $\mathbf{A}$ is an $N \times K$ real matrix whose unit energy columns are signature vectors of K users, $\mathbf{W} = diag(w_1, \cdots, w_K)$ is a diagonal matrix of users' amplitudes, $\mathbf{d}$ is a $K \times 1$ vector whose elements are users' binary symbols, and $\mathbf{n}$ is an $N \times 1$ background noise vector. Here, N is the processing gain, $\mathbf{d} = [d_1 \ \cdots \ d_K]^T$, where $d_k \in \{+1, -1\}$ denotes the kth user binary symbol, $n_i = [\mathbf{n}]_i$ is additive white Gaussian noise with zero mean and σ^2 variance. Note that $a_{i,k} = [\mathbf{A}]_{i,k} = \{\pm 1/\sqrt{N}\}$.

Suppose that user 1 is the desired user. Eq. (1) can be rewritten as

$$\mathbf{y} = \mathbf{a}_1 w_1 d_1 + \bar{\mathbf{A}}_1 \bar{\mathbf{W}}_1 \bar{\mathbf{d}}_1 + \mathbf{n}, \tag{2}$$

where $\mathbf{a}_k$ ($\mathbf{w}_k$) is the kth column vector of $\mathbf{A}$ (resp., $\mathbf{W}$). Here, $\bar{\mathbf{A}}_k = [\mathbf{a}_1 \ \cdots \ \mathbf{a}_{k-1} \mathbf{a}_{k+1} \ \cdots \ \mathbf{a}_K]$, $\bar{\mathbf{W}}_k = diag(w_1, \cdots, w_{k-1}, w_{k+1}, \cdots, w_K)$, and $\bar{\mathbf{d}}_k = [d_1 \ \cdots \ d_{k-1} \ d_{k+1} \ \cdots \ d_K]^T$. In general, $\bar{\mathbf{A}}_{k_1,\ldots,k_m}$ denotes the matrix which is obtained from $\mathbf{A}$ by deleting the column vectors corresponding to the indices $k_1, \ldots, k_m$. In addition, we define $\bar{\mathbf{W}}_{k_1,\ldots,k_m}$ which is obtained from $\mathbf{W}$ by deleting the diagonal elements corresponding to the indices $k_1, \ldots, k_m$. The vector denoted by $\bar{\mathbf{d}}_{k_1,\ldots,k_m}$ is also obtained from $\mathbf{d}$ by deleting the elements of the indices $k_1, \ldots, k_m$. Assume that d_1 is given. Then, the unconstrained maximum-likelihood estimate of $\bar{\mathbf{u}}_1 = \bar{\mathbf{W}}_1 \bar{\mathbf{d}}_1$ is written as

$$\hat{\bar{\mathbf{u}}}_1 = (\bar{\mathbf{A}}_1^T \bar{\mathbf{A}}_1)^{-1} \bar{\mathbf{A}}_1^T \mathbf{v}_1, \tag{3}$$

where $\mathbf{v}_1 = \mathbf{y} - \mathbf{a}_1 w_1 d_1$. Substituting (3) into (2) yields the detector for d_1 as

$$\hat{d}_1 = \arg \min_{d_1 \{+1,-1\}} ||\mathbf{M}_1(\mathbf{y} - \mathbf{a}_1 w_1 d_1)||^2, \tag{4}$$

where $\mathbf{M}_1$ is the projection matrix and given as $\mathbf{M}_1 = \mathbf{I} - \bar{\mathbf{A}}_1(\mathbf{A}_1^T \mathbf{A}_1)^{-1} \bar{\mathbf{A}}_1^T$. This detector is called the multiuser projection detector and has been proposed in [3].

In above, we consider an implementation of the projection detector. In order to find the projection matrix, a $(K-1) \times (K-1)$ matrix inversion is required.

3 An approximate matrix inversion for projection matrix

It is known that the projection detector in (4) provides the same bit error rate (BER) performance as the decorrelating detector for uncoded data sequences, but it provides better performance for coded sequences [3]. A common difficulty of the decorrelating detector and projection detector is computational complexity for matrix inversion. To compute $\mathbf{M}_1$, a matrix inversion of a $(K-1) \times (K-1)$ matrix per bit is required (we assume long-code spreading). Similarly, an inversion of $K \times K$ matrix per bit is also required for the decorrelating detector. Since the computational complexity of matrix inversion has order $O(K^3)$ per bit, the implementation of those multiuser detector would be difficult in some applications.

In [2], a variation of the decorrelating detector has been proposed to reduce computational complexity using an approximate matrix inversion. For the multiuser projection detector, the same approximate matrix inversion can be also applied to reduce computational complexity.

Suppose that $\mathbf{F}$ is an $n \times n$ matrix and $||\delta \mathbf{F}|| < 1$, where δ is a scalar. Then,

$$(\mathbf{I} - \delta \mathbf{F})^{-1} = \sum_{\ell=0}^{\infty} (\delta \mathbf{F})^\ell.$$

Hence, $(\mathbf{I} - \delta \mathbf{F})^{-1} = \mathbf{I} + \delta \mathbf{F} + o(\delta)$. This provides an approximation

$$(\bar{\mathbf{A}}_1^T \bar{\mathbf{A}}_1)^{-1} \simeq 2\mathbf{I} - \bar{\mathbf{A}}_1^T \bar{\mathbf{A}}_1. \tag{5}$$

This approximate inverse can be applied to compute an approximation of $\mathbf{M}_1$, i.e.,

$$\mathbf{M}_1 \simeq \hat{\mathbf{M}}_1 = \mathbf{I} - \bar{\mathbf{A}}_1(2\mathbf{I} - \bar{\mathbf{A}}_1^T \bar{\mathbf{A}}_1)\bar{\mathbf{A}}_1^T. \tag{6}$$

Hence, the matrix inversion of order $O((K-1)^3)$ per bit is not required. In addition, the resulting multiuser projection detector can be readily implemented as the approximate decorrelating detector in [2].

Although the approximation in (6) provides a computationally efficient approximate multiuser projection detector, the performance depends on the accuracy of $\hat{\mathbf{M}}_1$. Generally, if there are columns of which cross-correlations are high, the accuracy of $\hat{\mathbf{M}}_1$ cannot be reasonable. To provide better accuracy, we can consider a generalization of (2).

Suppose that the column vectors of $\bar{\mathbf{A}}_1$ for (k_1, k_2) users are highly correlated. Let $\mathbf{A}_c \triangleq [\mathbf{a}_1 \ \mathbf{a}_{k_1} \ \mathbf{a}_{k_2}]$. In addition, let $\mathbf{A}_u = \bar{\mathbf{A}}_{1,k_1,k_2}$. Then, Eq. (2) can be rewritten as

$$\mathbf{y} = \mathbf{A}_c \mathbf{W}_c \mathbf{d}_c + \mathbf{A}_u \mathbf{W}_u \mathbf{d}_u + \mathbf{n}, \qquad (7)$$

where $\mathbf{W}_c = diag(w_1, w_{k_1}, w_{k_2})$, $\mathbf{W}_u = \bar{\mathbf{W}}_{1,k_1,k_2}$, $\mathbf{d}_c = [d_1 \ d_{k_1} \ d_{k_2}]^T$, and $\mathbf{d}_u = \bar{\mathbf{d}}_{1,k_1,k_2}$. The resulting detector for $\mathbf{d}_c$ is written as

$$\hat{\mathbf{d}}_c = \arg \min_{\mathbf{d}_c \in \{+1,-1\}^3} ||\mathbf{M}_u(\mathbf{y} - \mathbf{A}_c \mathbf{W}_c \mathbf{d}_c)||^2, \quad (8)$$

where $\mathbf{M}_u = \mathbf{I} - \mathbf{A}_u(\mathbf{A}_u^T \mathbf{A}_u)^{-1} \mathbf{A}_u^T$. Since the two column vectors in $\bar{\mathbf{A}}_1$ for users (k_1, k_2) are removed, the approximation

$$(\mathbf{A}_u^T \mathbf{A}_u)^{-1} \simeq 2\mathbf{I} - \mathbf{A}_u^T \mathbf{A}_u$$

would be better than $(\bar{\mathbf{A}}_1^T \bar{\mathbf{A}}_1)^{-1} \simeq 2\mathbf{I} - \bar{\mathbf{A}}_1^T \bar{\mathbf{A}}_1$. An approximate of $\mathbf{M}_u$ is written as

$$\hat{\mathbf{M}}_u = \mathbf{I} - \mathbf{A}_u(2\mathbf{I} - \mathbf{A}_u^T \bar{\mathbf{A}}_u)\bar{\mathbf{A}}_u^T. \qquad (9)$$

This approximation can be used to reduce computational complexity for the detector in (8).

In (8), $2^3 = 8$ metrics shall be computed. This slightly increases computational complexity. Once we compute $\mathbf{M}_u \mathbf{y}$ and $\mathbf{M}_u \mathbf{A}_c \mathbf{W}_c$ or their approximates, we only need vector addition or subtraction operations to find 8 metrics. It does not significantly increase the computational complexity.

Two users whose column vectors are highly correlated can be found as follows: choose the two largest absolute values among the elements $[\mathbf{A}^T \mathbf{A}]_{m,n}$ for $m = 2, \ldots, K$, $n > m$. The corresponding indices are (k_1, k_2) which are the new two indices for 3-constrained users detection in (8). Hence, the approximate matrix inverse can have better accuracy by removing highly correlated two columns, and this can improve the approximation.

4 Simulation results

In simulations, we consider Gold codes for the spreading codes and the processing gain, N, is set to 31.

Simulation results are presented in Figures 1 and 2. In the legends of the plots, "1C-PR" and "1C-PR with approx." denote the detector for one constrained user detection in (3) and its variation with the approximate inverse in (6), respectively. Similarly, "3C-PR" and "3C-PR with approx." denote the detector for 3-constrained users detection in (8) and its variation in (9), respectively. For performance comparison, the decorrelating detector and approximate decorrelating detector are taken into account. In the legend, those are represented by "DC" and "Approx. DC", respectively. The conventional detector with correlator-matched filter is also considered (in the legend, it is represented by "matched").

Figure 1 shows BER performance for six detectors with various values of signal to noise ratio (SNR). The SNR is defined as $1/\sigma^2$ and all amplitudes are set to 1, i.e., $w_k = 1$ for all k. The performance of the multiuser projection detector is the same as the decorrelating detector as shown in [3]. The performance of 1-constrained user multiuser projection detector with $\hat{\mathbf{M}}_1$ is quite close to the multiuser projection detector, especially, for lower SNR. If the number of constrained users for detection gets larger, the performance of the multiuser projection detector gets better. According to Figure 1, the performance of 3-constrained user multiuser projection detector with $\hat{\mathbf{M}}_u$ is almost the same as that of 3-constrained user multiuser projection detector.

In Figure 2, simulation results are shown for various number of K. It is shown that when the CDMA system is over-loaded (e.g., $K/N > 1/2$), the performance of the detectors based on approximate matrix inverses is not satisfactory. On the other hand, if the CDMA system is not over-loaded, the approximate projection can provide reasonable performance while reducing computational complexity.

The accuracy of the approximate matrix inverse can be improved by removing highly correlated users. Of course, if there are more than two highly correlated users (this can happen when a CDMA system becomes over-loaded, i.e., K gets larger), the performance may not be satisfactory by just removing the two most correlated users as shown in Figure 2. For larger K, more users whose column vectors are highly correlated shall be included in the constrained user set. However, this may increase complexity in computing metrics.

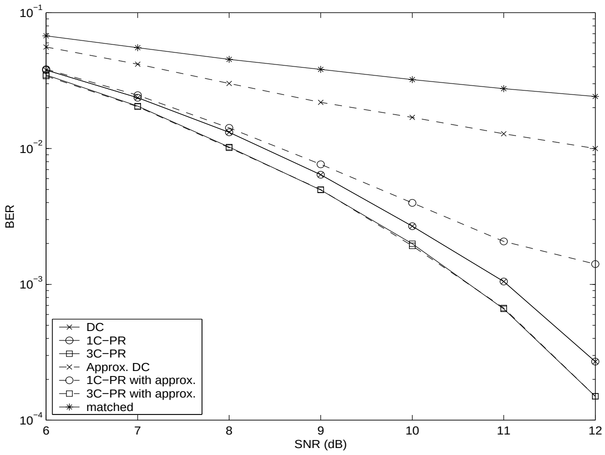

Figure 1: BER performance for six detectors with $N = 31$ and $K = 7$.

5 Conclusions

In the paper, we consider an approximate projection detector. In order to reduce the computational complexity, an approximation for the matrix inverse has been considered. By choosing highly correlated users, we can reduce the performance degradation due to the approximation of the matrix inverse. It is shown that the performance is satisfactory when the system is not over-loaded.

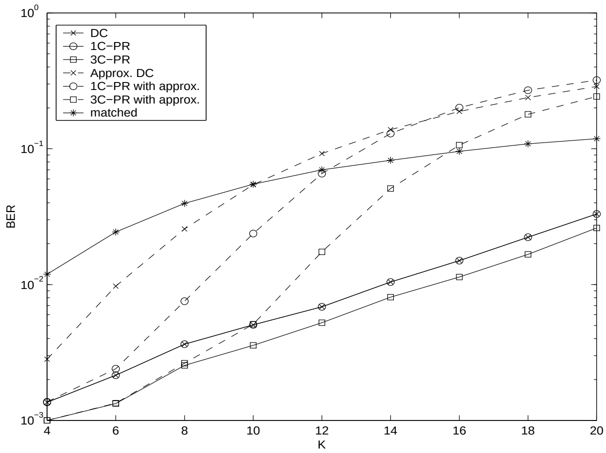

Figure 2: BER performance for six detectors with $N = 31$ and SNR = 10 dB.

References

[1] R. Lupas and S. Verdu, "Linear multiuser detectors for synchronous code-division multiple-access channels," *IEEE Tr. Infor. Theory*, vol.IT-34, pp.123-136, January 1989.

[2] N.B. Mandayam and S. Verdu, "Analysis of an approximate decorrelating detector," *Wireless Personal Comm.*, vol.6, No.1/2, pp.97-111, January 1998.

[3] C. Schlegel, P. Alexander, S. Roy, and Z. Xiang, "Multi-user projection receivers," *IEEE Jr. Selec. Areas Comm.*, vol.SAC-14, No.8, pp.1610-1618, October 1996.

[4] S. Verdu, *Multiuser Detection*, Cambridge University Press, New York, NY, 1998.