The Performance of Overlap PCC-OFDM with Error-Correcting Codes

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Abstract: Polynomial cancellation coded orthogonal frequency division multiplexing with overlapping symbol periods (Overlap PCC-OFDM) is a modulation technique that is less sensitive to frequency errors and more robust against multipath distortion than OFDM. A cyclic prefix is no longer needed. However, fading is still a problem and will cause bit errors. Error propagation will occur as decision feedback equalizer is used. In previous studies this was not taken into account and perfect decisions were assumed. In Overlap PCC-OFDM because the equalizer output is a vector rather than a scalar, error correction can be performed across the vector before the decision is fed back. In this paper, we apply error-correcting codes across the subchannels to improve the symbol error rate (SER). Comparisons are made between the SER performance of Overlap PCC-OFDM and OFDM in frequency error and in a two-path channel with both employing Reed-Solomon (RS) codes. Results show that Overlap PCC-OFDM outperforms OFDM.

1. Introduction

Orthogonal frequency division multiplexing (OFDM) is a modulation technique used in many new digital data transmission systems such as digital video broadcasting (DVB), digital audio broadcasting (DAB) and wireless area networks. However, OFDM suffers from sensitivity to frequency errors. The frequency errors due to Doppler effects and frequency offsets between the carrier frequency at the transmitter and the receiver degrades the performance of OFDM [1]. To eliminate intercarrier interference (ICI) arising from intersymbol interference (ISI) in multipath channel, OFDM employs a cyclic prefix, which results in the loss of spectral efficiency. Recently, a new technique called polynomial cancellation coded OFDM (PCC-OFDM) has been proposed to improve the performance of OFDM against frequency errors and to increase its tolerance to multipath with large delay spread [2].

PCC-OFDM is an OFDM technique that maps data to be transmitted onto weighted groups of subcarriers instead of individual subcarriers. The mapping of data onto the weighted groups of subcarriers results in the flattening of the frequency domain sidelobes of the subcarriers and hence makes PCC-OFDM less sensitive to frequency offset and Doppler spread [3, 4]. The mapping also causes the energy of the transmitted symbol to be concentrated in the middle of the symbol period. This will make PCC-OFDM more robust against the interference caused by the multipath propagation or timing effect [2].

The advantages of PCC-OFDM have been obtained at the cost of reduced data rate. In its simplest form, the spectral efficiency of PCC-OFDM is approximately half that of normal OFDM. The elimination of cyclic prefix, reduction in ICI and improved spectral properties in PCC-OFDM would make up some of the loss but not all. The authors in [2] have shown that the advantages of PCC-OFDM can be retained without any loss in spectral efficiency by overlapping the symbols in the time domain and the data can be recovered at the receiver using a two dimensional equalizer.

The performance of Overlap PCC-OFDM is comparable with normal OFDM in AWGN while it outperforms OFDM in multipath transmission [5]. In [5] it was assumed that the correct decisions were fed back to the equalizer at the receiver to recover overlapped symbols. This is not possible in a practical system. However error-correcting codes can be applied so that the most of the errors can be corrected and only a few residual errors will propagate through the equalizer.

In this paper, the performance of Overlap PCC-OFDM is presented with the error correcting codes employed across each

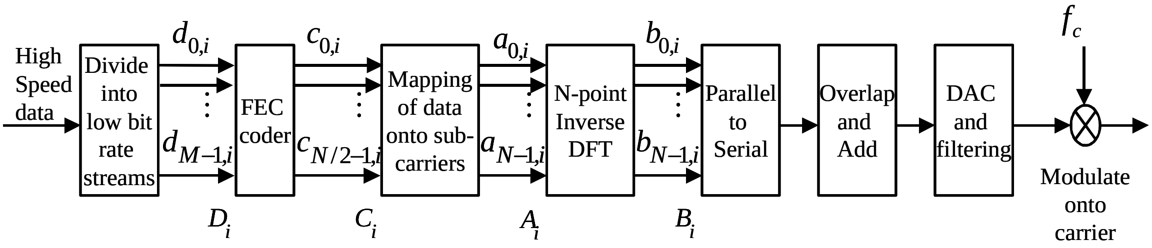

Figure 1: The transmitter of Overlap PCC-OFDM

symbol. The results are compared with that of OFDM when the multipath delay exceeds the length of cyclic prefix. The simulations are performed in additive white Gaussian noise (AWGN) channel with frequency offset as well as in a two-path channel. Reed-Solomon codes with the message block length of 11 and the code block length of 15, $RS(15{,}11)$ are employed. Reed-Solomon codes are preferred as they are very good at correcting bursts of errors that results in fading environment.

2. PCC-OFDM communication systems with overlapping symbol periods

Figure 1 shows the transmitter for an Overlap PCC-OFDM communications system. High speed incoming serial data are converted to parallel substreams. Error-correcting codes are applied on data vector $D_i = [d_{0,i}, \cdots, d_{n-1,i}]$ that represents the data to be transmitted in i-th symbol period. The outputs of the FEC coder are given by $c_{0,i}, \cdots, c_{n-1,i}$. In normal OFDM, $n = N$ and each $c_{k,i}$ is mapped onto individual subcarriers. The number of subcarriers is N. However in PCC-OFDM, $c_{k,i}$ is mapped onto weighted groups of adjacent subcarriers. In our study mapping of data onto pairs of subcarriers is considered so that we have $n = N/2$. The mapping rule is $a_{2k,i} = c_{k,i}$, $a_{2k+1,i} = -c_{k,i}$ for $1 \le k \le N/2 - 1$. The term ‘subchannels’ will be used to describe weighted pairs of subcarriers. The outputs of inverse DFT, $b_{0,i}, \cdots, b_{N-1,i}$ are converted parallel to serial. In simple PCC-OFDM, the discrete baseband signal is then converted to analogue and filtered before being modulated onto a carrier frequency. However in Overlap PCC-OFDM, the data of the i-th symbol period is overlapped with half of the $(i-1)$-th symbol and also half of the $(i+1)$-th symbol in ‘add and overlap’ block before being converted digital-to-analogue. The rate of inverse DFT clocking is increased to perform one transform every $T/2$ period, where T is the symbol period.

Figure 2 shows the form of the overlapped symbols. The overlapping period of $T/2$ is considered in this paper. The shape of each symbol is chosen to highlight the distribution of energy within a symbol. The overlapping of symbols introduces ISI at the transmitter.

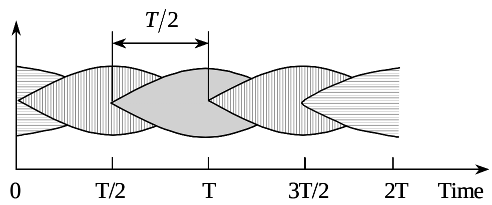

Figure 2: Overlap PCC-OFDM symbols

Figure 3 shows the block diagram of the receiver in an Overlap PCC-OFDM system. The received signal is filtered, down-converted into baseband signal and converted to give discrete samples. Input samples of length N to DFT are obtained by moving the window by $N/2$ samples every $T/2$. The DFT demodulator outputs in the i-th symbol periodare given by $Z_i = [z_{0,i}, \cdots, z_{N-1,i}]$ that depends mainly on the input vector $A_i = [a_{0,i}, \cdots, a_{N-1,i}]$ and partly on the adjacent input vectors A_{i-1} and A_{i+1}. The vector Z_i is input to the ‘weighting and adding’ block that operates as a matched filter to PCC waveforms to output vector V_i and further contributes to ICI cancellation properties of the technique [6]. The outputs of the ‘weighting and adding’ block are then fed to a two-dimensional equalizer to recover the transmitted data sequence. Finally the FEC decoder will work on the outputs of the equalizer to correct errors before the error corrected symbols are fed back.

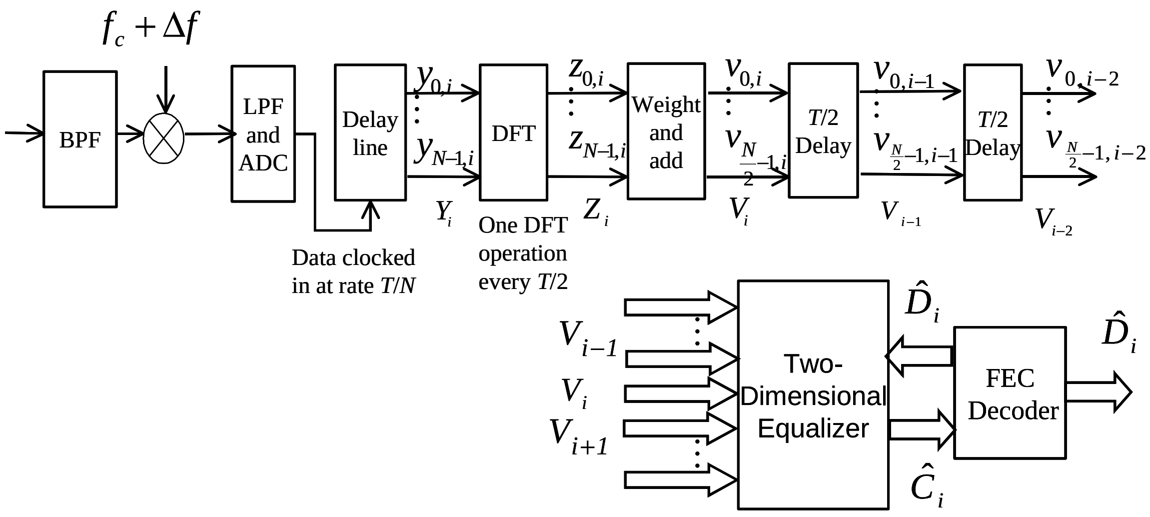

Figure 3: The receiver of Overlap PCC-OFDM

The equalizer operates at the output of the DFT and has to combat interference that occurs between symbols in the time domain as well as interference between subchannels in the frequency domain. Because of the ISI/ICI cancellation properties of PCC, there are few significant tap values and even the severe multipath distortion has little effect on the number of significant terms [2]. The equaliser used in this study is a decision feedback equalizer (DFE) with minimum mean square error (MMSE) feedforward section. The MMSE equalizer taps were calculated taking into account the correlated noise. Four feedforward stages and one feedback stage were used. Estimates of transmitted data in the i-th symbol period are recovered using input vector V_{i-1} to V_{i+4}. The DFE complexity can be reduced by using only three taps in each stage without noticeable degradation in the performance of the system [5].

3. Error-correcting codes in Overlap PCC-OFDM

In [2, 5] it was assumed that the correct decisions are fed back to the equalizer. In practice, correct decisions will not be available at the receiver. Despite its improved robustness against delays in multipath, fading is still a problem for Overlap PCC-OFDM. Fading will cause errors in the data estimates at the receiver and the errors in any symbol will propagate through to the other symbols due to the feedback in the equalizer. This will degrade the performance of the system. Therefore, forward error correction (FEC) is essential to combat the effect of multipath fading and frequency errors. The error-correcting codes that are applied to OFDM are also applicable to Overlap PCC-OFDM. Studies have shown that FEC with appropriate channel interleaving techniques is needed to combat the burst of errors that are caused by frequency selective fading [7]. Most wireless application of OFDM use a coding scheme, called concatenated coding that combines Reed-Solomon codes with convolutional codes with appropriate channel interleavers [7, 8]. Recent studies have also considered the use of trellis codes [9] and turbo codes [10] to improve on the error rate. Often coding schemes incorporate puncturing of codes to increase their code rates.

The application of error-correcting codes in Overlap PCC-OFDM requires some additional considerations. Since error corrected symbols have to be fed back as input vectors to remove ISI introduced at the transmitter, the codes have to be applied across the subchannels in each symbol. At the receiver, decoding is also applied across the subchannels in each vector output of equalizer. Interleavers also have to work within each symbol. The choice of interleaving technique as well as the power of the codes depends on the size of the vector, that is the number of subchannels in the system.

The aim of this paper is not to apply the best coding scheme available. The aim here is to apply a simple coding scheme to both OFDM and Overlap PCC-OFDM, and to compare their respective performances in frequency errors and in multipath. The Reed-Solomon codes $RS(15,11)$ have been used. Simulations have been obtained for 16-QAM symbols, $N = 128$ and the number of subchannels in PCC-OFDM is 64.

4. Simulation results

In all simulations, a cyclic prefix of $30(T/128)$ is considered in OFDM except for the case when the cyclic prefix is long enough to absorb the delay of length $35(T/128)$. Energy per bit to noise ratio E_b/N_0 in the transmitted signal is normalized to unity. Transmitted signal energy is also normalized for the loss of energy due to the use of cyclic prefix in OFDM and due to the mapping of each data symbol onto two subcarriers in Overlap PCC-OFDM. A two-path channel is used with $3/4$ of signal power in the delayed path.

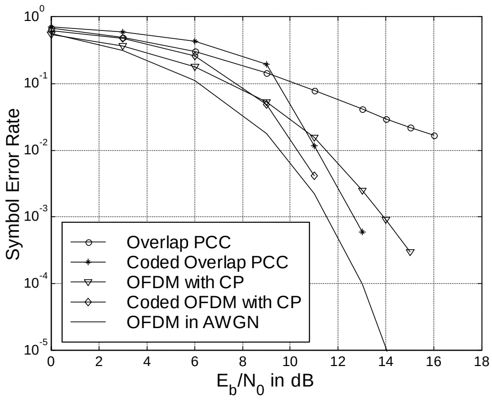

Figure 4: SER performance in AWGN with $\Delta fT = 0.0$

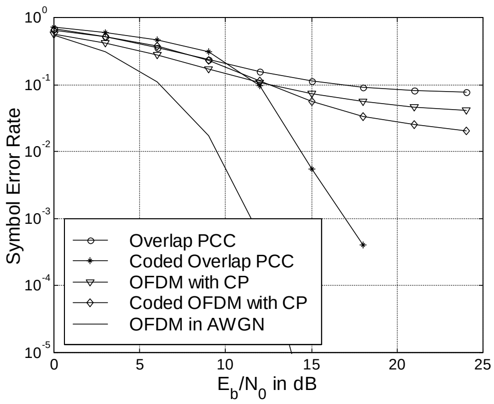

Figure 5: SER performance in AWGN with $\Delta fT = 0.10$

Figure 4 shows the SER performance of OFDM and Overlap OFDM in AWGN channel with no frequency errors. The SER of uncoded OFDM is slightly higher than the theoretical plot due to energy used by cyclic prefix. When $RS(15,11)$ is applied, Overlap PCC-OFDM has a similar performance to OFDM. However the advantage of using Overlap PCC-OFDM will become clear in frequency errors and in multipath when delay is longer than the cyclic prefix. Figure 5 shows the SER of OFDM and Overlap PCC-OFDM in the presence of normalized frequency offset $\Delta fT = 0.10$. The SER of uncoded Overlap PCC-OFDM is slightly higher than that of OFDM. This is due to the error propagation in the DFE equalizer. When the error corrected decisions are fed back to the equalizer by applying $RS(15,11)$ across the subchannels, the SER of Overlap PCC-OFDM improves and outperforms OFDM with $RS(15,11)$. The SER plot of Overlap PCC-OFDM with $RS(15,11)$ decays very rapidly above $12\,\text{dB}$ whereas SER of OFDM remains flat. The high sensitivity of OFDM to frequency errors results in high ICI that degrades the performance of OFDM.

Plots of the SER for Overlap PCC-OFDM and OFDM are presented in figure 6. The delay in the echo path is $35(T/N)$ long and exceeds the length of cyclic prefix. The performance of OFDM degrades when delay is longer than cyclic prefix. The SER of Overlap-OFDM is lower than that of OFDM. The figure also shows the performance of Overlap PCC-OFDM is further improved when $RS(15,11)$ is applied and error corrected decisions are fed back to the equalizer. The SER of Overlap PCC-OFDM begins to decay rapidly when E_b/N_0 exceeds $15\,\text{dB}$ whereas the SER of OFDM remains still higher and improvement due to error-correcting code is small. This is because cyclic prefix is not long enough to absorb all of the delay and the delay spread will cause ICI. The plot for OFDM when the cyclic prefix absorbs all of the delay is also shown in figure 6 showing the SER starts decaying rapidly with increasing E_b/N_0. However, at higher E_b/N_0, Overlap PCC-OFDM has lower SER. The reason for higher SER of Overlap PCC-OFDM at lower E_b/N_0 is due to error propagation of residual errors in the equalizer.

The SER performance can be further improved by using more powerful coding technique. In a fading channel, OFDM employs concatenated coding scheme that combines multiple error correcting Reed-Solomon codes and $1/2$-rate convolutional codes with appropriate channel interleaving techniques. With the use of such schemes, the data transmission in Overlap PCC-OFDM can be made almost error free. Figure 7 show the plots for Overlap PCC-OFDM, the SER decays rapidly above 10 dB of E_b/N_0 when $RS(15,11)$ is applied across the subchannels and error free symbols are fed back to the equalizer.

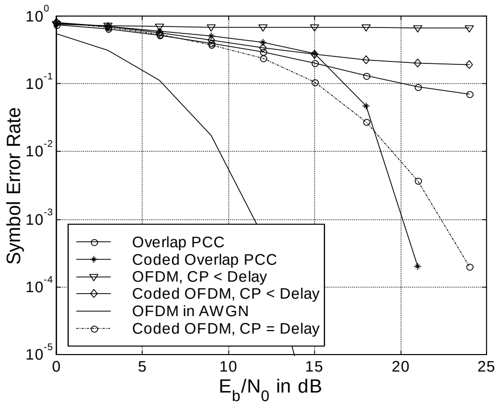

Figure 6: SER performance in 2-path channel, error corrected decisions fed back

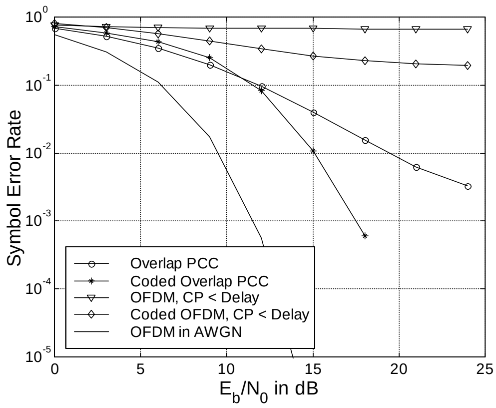

Figure 7: SER performance in 2-path channel, correct decisions fed back

5. Conclusions

PCC-OFDM with overlapping symbol periods is less sensitive to frequency errors and more robust against delays in multipath than OFDM. In this paper, we have shown that by applying error-correcting codes across subchannels in each symbol the SER of the PCC-OFDM with overlapping symbol periods can be further reduced. Most of the coding schemes that have been considered for OFDM in fading channel are applicable to Overlap PCC-OFDM. However the error-correcting codes and the interleaving techniques have to be employed within each symbol. More powerful coding schemes can be used to further improve the performance of Overlap PCC-OFDM.

6. References

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