Analysis and Synthesis of Three Dimensional Carrierless AM/PM System

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Abstract

In this paper, a new approach is introduced for designing three dimensional carrierless AM/PM system. This approach is based on the technique used for transmultiplexer system design. The original continuous time three dimensional carrierless AM/PM system design problem is first converted to a digital transmultiplexer system design problem, and then it is divided into two optimization problems, one for transmitter design, another for receiver design. Finally, the transmitter design problem is converted to a minimax optimization problem and the receiver design problem is converted to a quadratic programming, respectively. Compare with the existed methods, it is shown that this approach is efficient and significant.

1. Introduction

Carrierless AM/PM (CAP) is a bandwidth efficient two dimensional (2D) passband transmission scheme, which is closely related to the more familiar quadrature amplitude modulation (QAM) transmission scheme [1, 2]. The CAP transceivers are particularly well suitable to all digital implementations for applications where the bandwidth of the signal's spectrum is of the same order of magnitude as the center frequency of the spectrum. As a major application, the 16-CAP system has been chosen as the asynchronous transfer mode (ATM) local area network physical layer interface standard for category 3 unshielded twisted pair wiring at 51.84 Mb/s by the Technical Committee of the ATM Forum [2].

In order to improve the system channel efficiency, recently a new technique of extending the original two dimensional CAP system to a three dimensional (3D) CAP system has been introduced in [3]. This idea offers many opportunities for improvement over the traditional CAP system, one obvious advantage is to increase the system throughput at the cost of increased complexity and increased receiver interference energy. Another possible application is in the area of the multiple access communication for the digital communication environment, this will allow multiple users to enjoy their own channels of communications while they use the same physical communication link.

2. Conversion of 3D CAP system into Transmultiplexer

The conventional three dimensional CAP system structure is shown in Fig. 1. The data stream to be transmitted is first encoded into three symbol sequences, and then each symbol sequence is fed into the corresponding pulse shaping filter (signature waveform). The output signals of three shaping filters are simultaneously passed through the system channel. The signal $s(t)$ at the output of the CAP transmitter can be written as:

$$s(t) = \sum_{n=-\infty}^{\infty} \Big(s_0(n) f_0(t - nT) + s_1(n) f_1(t - nT) + s_2(n) f_2(t - nT) \Big) \tag{1}$$

where T is the symbol period, $\{s_0(n)\}$, $\{s_1(n)\}$ and $\{s_2(n)\}$ are discrete multilevel symbol sequences which are sent in symbol period nT. $f_0(t)$, $f_1(t)$ and $f_2(t)$ are the impulse response of the shaping filters, respectively [1][3].

Originally, the pulse shaping filters were designed as analogue filters [4], due to the difficulty of their realization, they are replaced by the digital filters. Actually, the analogue pulse shaping filters $\{f_0(t)$, $f_1(t)$, $f_2(t)\}$ are sampled at a rate which satisfies the Nyquist criterion for the given bandwidth, the input symbol sequence is upsampled to match the pulse shaping filter sampling rate as well. The actual transmitted signal $s(t)$ is generated by feeding the output through a DAC and a reconstruction filter. In receiver, the signal is filtered by an antialiasing filter and an ADC, and then passes through three different receiver filters which have the same sampling rate as the transmitter filters, finally they are downsampled to the original

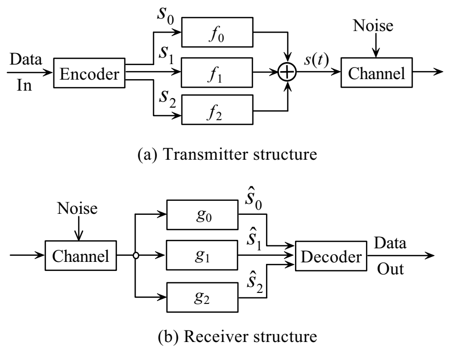

Figure 1: Block diagram of 3D CAP system

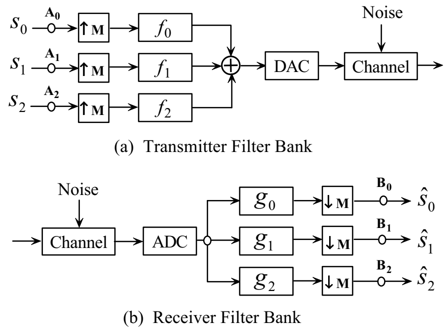

Figure 2: Three dimensional transmultiplexer structure

symbol rate. This model is similar to the transmultiplexer system displayed in Fig. 2. In the ideal channel case, the input symbol sequences $\{s_i(n)\}$ should be perfectly retrieved at the receiver, in other words, the output symbol sequence $\{\hat{s}_j(n)\}$ at the jth receiver should satisfy the condition:

$$\hat{s}_j(n) = s_j(n - n_0); \qquad j = 0, 1, 2. \tag{2}$$

where n_0 is a time delay.

The 3D CAP system design problem posed in [3] can be stated as follows: *Find three digital transmitter filters with given frequency magnitude characteristic, subject to the condition that they form a perfect reconstruction system with three digital receiver filters.*

In filter bank theory [5], it has been proved that each subtransceiver system from point A_i to B_j in Fig. 2 is a linear time invariant (LTI) system whose system impulse response depends on the transceiver filter pair $\{f_i, g_j\}$ and the upsampling (downsampling) rate M. Assume its transfer function is $H_{ij}(z)$, then we have

$$H_{ij}(z) = E^M_{0(ij)}(z) \tag{3}$$

where $E^M_{0(ij)}(z)$ is the 0th Type 1 polyphase representation [5] of the transceiver filter bank with

$$\begin{aligned} E^M_{0(ij)}(z) &= \sum_{n=-\infty}^{\infty} e_{0(ij)}(n) z^{-n} \\ e_{0(ij)}(n) &= l_{ij}(nM); \quad n = 0, \pm 1, \pm 2, \cdots \\ l_{ij}(n) &= \sum_{k=-\infty}^{\infty} f_i(k) g_j(n-k); \quad i, j = 0, 1, 2. \end{aligned} \tag{4}$$

In the ideal channel case, the output signal $\{\hat{s}_j\}$ can be expressed as:

$$\hat{s}_j(z) = \sum_{i=0}^{2} s_i(z) H_{ij}(z); \qquad j = 0, 1, 2. \tag{5}$$

Since the input signal $\{s_0(n)\}$, $\{s_1(n)\}$ and $\{s_2(n)\}$ are independent of each other, in order to satisfy condition (2), the system transfer function $H_{ij}(z)$ must satisfy

$$H_{ij}(z) = \begin{cases} z^{-n_0} & i = j \\ 0 & i \neq j \end{cases} \tag{6}$$

Thus, the three dimensional transceiver system design problem stated before can be posed mathematically as follows:

$$\min_{\{f_0, f_1, f_2\}} \left\{ \max_i \{ \max_f (|\mathcal{F}_i(f) - \mathcal{D}(f)|) \} \right\} \tag{7}$$

$$\text{subject to } H_{ij}(z) = \begin{cases} z^{-n_0} & i = j \\ 0 & i \neq j \end{cases}$$

where f_i is the transmitter filter, $\mathcal{F}_i(f)$ is its frequency magnitude response, $\mathcal{D}(f)$ is the frequency magnitude response of a desired pulse shaping filter and n_0 is a time delay.

3. The System Analysis and Suboptimal Approach

The transceiver system design problem (7) presented in the previous section is a general case in which both FIR filter and IIR filter can be employed. In this paper, we only consider the situation of both transmitter and receiver being FIR filter. Suppose $\mathbf{f_i} = [f_i(0)\ f_i(1) \cdots f_i(N-1)]^T$ and $\mathbf{g_j} = [g_j(0)\ g_j(1) \cdots g_j(K-1)]^T$. According to (4), the optimization problem (7) is written as:

$$\min_{\{f_0, f_1, f_2\}} \left\{ \max_i \{ \max_f (\left| \mathcal{F}_i(f) - \mathcal{D}(f) \right|) \} \right\} \tag{8}$$

$$\text{subject to } e_{0(ij)}(n) = \begin{cases} 1 & n = n_0 \text{ and } i = j \\ 0 & \text{else} \end{cases}$$

Basically, if we let $\mathbf{x} = [\mathbf{f_0}^T\ \ \mathbf{f_1}^T\ \ \mathbf{f_2}^T]^T$, then $\max_f (\left|\mathcal{F}_i(f) - \mathcal{D}(f)\right|)$ is a function of vector $\mathbf{x}$, which can be also treated as an object function, thus the problem (8) is a general minimax optimization problem. Since the object function includes both maximizing operation and calculating modulus of a complex function which usually make the object function to be unsmooth. Furthermore, the constrained equations also include three unknown variables $\{\mathbf{g_0}, \mathbf{g_1}, \mathbf{g_2}\}$ which do not appear in the object function. These make it very difficult to solve this problem directly so that we have to consider some methods for an approximate solution. We know that the conventional 2D CAP system design process is completed by two steps. The two pulse shaping filters are designed first, and then the receiver filters. Apparently, we can apply this idea to our three dimensional transceiver system design problem. In other words, we can divide the original minimax optimization problem (8) into two optimization problems. One for the transmitter filter design, and another for the receiver filter design.

Thus, the problem we are facing will be how to divide the minimax optimization problem (8). Since the main idea of this approach is to design the transmitter and receiver filters separately instead of simultaneously. For the transmitter filter design, the condition of all three transmitter filters having the same desired frequency magnitude characteristic should therefore be kept, but we can not put the original constraint in problem (8) into the new design process. On the other hand, in order to avoid getting a trivial solution such as the three transmitter filters have the same values, some rational constraint are also necessary. Recalling the discussion about the pulse shaping filter design in [3]. It has been pointed out that the desired pulse shaping filters should have three properties of achieving bandwidth efficiency, zero ISI and zero CSI, where the property of zero CSI is concerned of two different filters, so it should be logical to put this property as a constraint. We know [6] one usual idea of achieving zero CSI is that all the transmitter filters are orthogonal to each other in time domain, as such the transmitter filter design problem can be formulated as the following optimization problem:

$$\min_{\{f_0,f_1,f_2\}} \left\{ \max_i \left\{ \max_f |\mathcal{F}_i(f) - \mathcal{D}(f)| \right\} \right\} \tag{9}$$
$$\text{subject to} \quad \langle \mathbf{f_i}, \mathbf{f_j} \rangle = 0; \quad i \neq j. \; i,j = 0,1,2.$$

where $\langle\cdot\rangle$ denotes inner product of two vectors.

When we obtain the transmitter filters by solving the optimization problem (9), the next problem will be how to design the receiver filters. We know as a widely accepted criterion of system performance, it is desired that a maximal SNR can be obtained at the output of receiver. Since the power of output noise at receiver is proportional to the power of receiver filter $\{g_j\}$, the maximal SNR should be obtained by minimizing the power of receiver filter $\{g_j\}$. Thus, the receiver filter design problem can be formulated as:

$$\min_{g_j} \|\mathbf{g_j}\|^2 \tag{10}$$
$$\text{subject to} \quad e_{0(ij)}(n) = \begin{cases} 1 & n = n_0 \text{ and } i = j \\ 0 & \text{else} \end{cases}$$

where

$$\|\mathbf{g_j}\|^2 = \sum_{k=0}^{K-1} |g_j(k)|^2$$

By now, the transceiver system design problem (8) has been divided into two optimization problem successfully. Obviously, we can obtain an approximate solution of the problem (8) by solving the optimization problems (9) and (10). This approach has two advantages over the method used in [3]. Firstly, the transmitter filter design does not depend on the receiver filter which provides us more choices for the desired transmitter filter, although they are constrained to be orthogonal to each other, it is consonant with the criterion of zero CSI appeared in the receiver design process. Secondly, for the receiver filter design, the criterion of maximizing SNR is included in our design process which is not included in the original problem posed in [3].

4. Design of Transmitter and Receiver Filter

In this section, we will deal with the corresponding algorithms to solve the transmitter and receiver filter design problems presented in the previous section.

4.1. Transmitter Filter Design

As stated previously, the object function in the transmitter filter design problem (9) is unsmooth which usually increase the degree of difficulty to solve the problem, hence it is necessary to make suitable conversion so that the problem can be somewhat easy to solve. Suppose $F_i(f)$ be the frequency response of transmitter filter f_i with

$$F_i(f) = \sum_{n=0}^{N-1} f_i(n) e^{-j2\pi f n} = \mathbf{f_i}^T \mathbf{h}(f); \quad i = 0,1,2.$$

then we have

$$\mathcal{F}_i(f) = \Big(F_i(f)F_i^*(f)\Big)^{1/2} = \Big(\mathbf{f_i}^T \mathbf{H}(f)\mathbf{f_i}\Big)^{1/2} \tag{11}$$

where * denotes complex conjugate and

$$\mathbf{h}(f) = \big[e^0 \; e^{-j2\pi f} \; \cdots \; e^{-j2\pi f(N-1)}\big]^T$$
$$\mathbf{h}^*(f) = \big[e^0 \; e^{j2\pi f} \; \cdots \; e^{j2\pi f(N-1)}\big]^T$$
$$\mathbf{H}(f) = \mathbf{h}(f)\mathbf{h}^*(f)^T$$

Notice that the function $\mathcal{F}_i(f)$ and $\mathcal{D}(f)$ are not negative, hence we can replace function $\mathcal{F}_i(f) - \mathcal{D}(f)$ in problem (9) by $\mathcal{F}_i^2(f) - \mathcal{D}^2(f)$ without any change of the problem's property. Similarly, the function $|\mathbf{f_i}^T \mathbf{H}(f)\mathbf{f_i} - \mathcal{D}^2(f)|$ can also be replaced by $\big(\mathbf{f_i}^T \mathbf{H}(f)$

$\mathbf{f_i} - \mathcal{D}^2(f)\big)^2$ without any change of the problem's solution. As such, we can solve the following optimization problem instead of problem (9):

$$\begin{aligned} &\min_{\{f_0,f_1,f_2\}} \max_i \Big\{ \max_f \big(\mathbf{f_i}^T \mathbf{H}(f)\mathbf{f_i} - \mathcal{D}^2(f)\big)^2 \Big\} \\ &\text{subject to } \langle \mathbf{f_i}, \mathbf{f_j} \rangle = 0;\ i \neq j. \end{aligned} \tag{12}$$

For given $\mathbf{f_i}$, the function $\big(\mathbf{f_i}^T\mathbf{H}(f)\mathbf{f_i} - \mathcal{D}^2(f)\big)^2$ is a continuous function of variable f on $[0, 2\pi]$, hence $\max_f \big(\mathbf{f_i}^T\, \mathbf{H}(f)\mathbf{f_i} - \mathcal{D}^2(f)\big)^2$ exists and is a function of variable $\mathbf{f_i}$ which indicates that the optimization problem (12) is a minimax optimization problem. Although we can treat three independent variables $\{\mathbf{f_0}, \mathbf{f_1}, \mathbf{f_2}\}$ as one new variable as mentioned in section 3 and solve this problem directly by using the existed algorithm [7], however, due to its particularity of object function $\max_f \big(\mathbf{f_i}^T\mathbf{H}(f)\mathbf{f_i} - \mathcal{D}^2(f)\big)^2$ only depending on the variable $\mathbf{f_i}$ for each fixed i, so it should be more efficient to solve this problem by the following steps:

Step 1. Solve the optimization problem:
$\min_{f_0} \Big\{ \max_f \big(\mathbf{f_0}^T\mathbf{H}(f)\mathbf{f_0} - \mathcal{D}^2(f)\big)^2 \Big\}$

Step 2. Substitute the solution $\mathbf{f_0}$ obtained in the previous step into the following optimization problem and then solve it.
$\min_{f_1} \Big\{ \max_f \big(\mathbf{f_1}^T\mathbf{H}(f)\mathbf{f_1} - \mathcal{D}^2(f)\big)^2 \Big\}$
subject to $\langle \mathbf{f_0}, \mathbf{f_1} \rangle = 0$.

Step 3. Substitute the solution $\mathbf{f_0}$ and $\mathbf{f_1}$ into the following optimization problem and then solve it.
$\min_{f_2} \Big\{ \max_f \big(\mathbf{f_2}^T\mathbf{H}(f)\mathbf{f_2} - \mathcal{D}^2(f)\big)^2 \Big\}$
subject to $\langle \mathbf{f_0}, \mathbf{f_2} \rangle = 0,\ \langle \mathbf{f_1}, \mathbf{f_2} \rangle = 0$

Compare the optimization problem included in each step above with (12), obviously they are more simple and easy to solve. In fact, we can simply discretize variable f over the interval $[0, 2\pi]$, and then treat polynomial function $\big(\mathbf{f_i}^T\mathbf{H}(f)\, \mathbf{f_i} - \mathcal{D}^2(f)\big)^2$ as a vector of object function, as such the optimization problems included in the processing steps above are three simple minimax problems which can be solved by using the minimax algorithm in [7].

4.2. Receiver Filter Design

The basic receiver filter design problem has been stated as an optimization problem (10) in the sense of maximizing SNR in the previous section. Suppose the impulse response of LTI subtransceiver system from point A_i to B_j be $\mathbf{h_{ij}}$ with $\mathbf{h_{ij}} = [h_{ij}(0)\ h_{ij}(1)\ \cdots\ h_{ij}(L-1)]^T$ where L is the length of vector $\mathbf{h_{ij}}$, then we have

$$h_{ij}(n) = e_{0(ij)}(n) = l_{ij}(nM); \tag{13}$$

where $e_{0(ij)}(n)$ and $l_{ij}(n)$ are defined in equation (4). According to (5), the output signal $\hat{s}_j$ can be expressed as:

$$\hat{s}_j = \sum_{i=0}^{2} s_i \otimes h_{ij}; \quad j = 0, 1, 2. \tag{14}$$

where $\otimes$ denotes convolution.

Suppose the impulse response of the combined subtransceiver system without upsampling and downsampling be $\mathbf{l_{ij}}$ with $\mathbf{l_{ij}} = [l_{ij}(0)\ l_{ij}(1)\ \cdots\ l_{ij}(P)]^T$ where $P = N + K - 1$, then

$$\mathbf{l_{ij}} = f_i \otimes g_j = \mathbf{F_i g_j} \tag{15}$$

where $\mathbf{F_i}$ is the P by K convolution matrix of transmitter filter f_i.

Combine equation (13) with (15), we have

$$\mathbf{h_{ij}} = \mathbf{F_i^{(M)} g_j} \tag{16}$$

where $\mathbf{F_i^{(M)}}$ is a submatrix of $\mathbf{F_i}$ with the Lth row vector of $\mathbf{F_i^{(M)}}$ being the $1 + (L-1)M$th row vector of matrix $\mathbf{F_i}$.

Now substitute (16) with (13) to (10), then the receiver filter design problem (10) is written as:

$$\begin{aligned} &\min_{g_j} \mathbf{g_j}^T \mathbf{g_j} \\ &\text{subject to} \quad \mathbf{F_i^{(M)} g_j} = \mathbf{d_{ij}}; \quad i, j = 0, 1, 2. \end{aligned} \tag{17}$$

where

$$\mathbf{d_{ij}} = \begin{cases} [\ \underbrace{0\ \cdots\ 0}_{n_0}\ 1\ 0\ \cdots\ 0\ ]^T; & i = j \\ [\ 0\ \cdots\ 0\ \ 0\ 0\ \cdots\ 0\ ]^T; & i \neq j \end{cases}$$

This is a quadratic problem with equality constraints which can be solved using the existing algorithm presented in [8][9] or Matlab function quadprog [7]. Furthermore, if there exists a solution to this problem, then the solution is the unique global solution.

The approach provided above is based on an ideal channel with zero ISI and CSI condition. One alternative formulation of problem (17) is to relax the constraints of zero ISI and CSI so that we can obtain a solution with lower receiver filter power corresponding to a higher SNR. Generally, since the channel's bandwidth is always limited in the practical system design process, it is impossible to achieve zero ISI and CSI indeed. Hence, it should be more significant to solve the following problem than (17).

$$\begin{aligned} &\min_{g_j} \mathbf{g_j}^T \mathbf{g_j} \\ &\text{subject to } \big|\mathbf{F_i^{(M)} g_j} - \mathbf{d_{ij}}\big| \leq \varepsilon; \quad i, j = 0, 1, 2. \end{aligned} \tag{18}$$

where ε is a given constant.

Certainly, it is also a quadratic optimization problem with inequality constraints. Hence, we can solve it using the algorithms mentioned above.

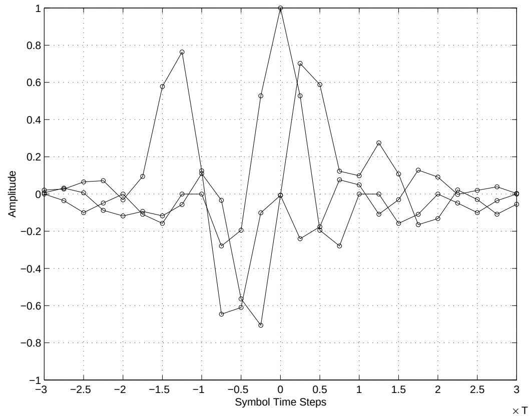

Figure 3: Three orthogonal transmitter filters $\{f_0, f_1, f_2\}$

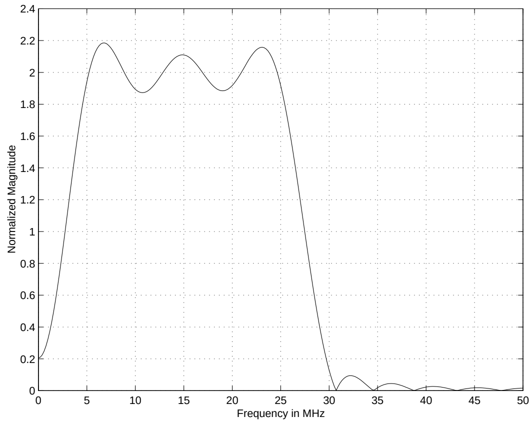

Figure 4: Transmitter filter's frequency response

5. Simulation Results

To project the effective performance of the approach proposed in this paper, a new 3-D transceiver system is presented by solving the problem (12) and (18). The three transmitter filters are displayed in Fig. 3, where the conventional raised-cosine filter is chosen as the desired shaping filter, and symbol rate of 25 MHz with excess bandwidth of 20% is used in this example [3]. The corresponding frequency response is displayed in Fig. 4. As we know, in order to eliminate the aliasing in the sampling process, the sampling rate must be not less than twice of the signal bandwidth $W = (1+\alpha)/T$. Therefore, the sampling point number within one time step T will be not less than $T \times 2W = 2(1+\alpha)$. Since $0 \leq \alpha \leq 1$, the choice of upsampling rate $M = 4$ will be suitable to any α. The filter length N is chosen as 25.

The receiver filters corresponding to the transmitter obtained previously is shown in Fig. 5. The receiver filter's length is chosen as $K = 65$, the constant $\varepsilon = 0.01$. The system time delay $n_0 = 2$. The system impulse response for each subtransceiver are shown in Figs. 6, 7 and 8. The simulation results show that this transceiver system performance is close to the desired system.

6. Conclusions

A new method has been provided for designing the 3D CAP system. This method divides the original 3D CAP system design problem into two optimization problems. The transmitter system design has been reformulated as a minimax optimization problem, and the receiver system design problem has been reformulated as a quadratic optimization problems.

7. References

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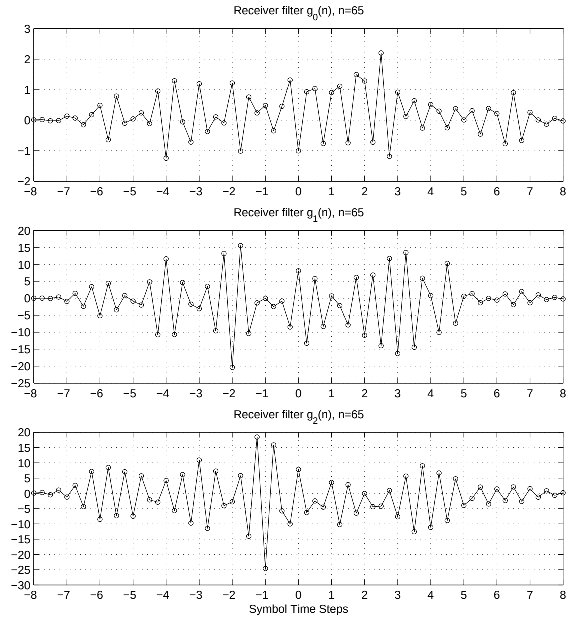

Figure 5: Three receiver filters $g_0(n)$, $g_1(n)$ and $g_2(n)$, $n = 65$.

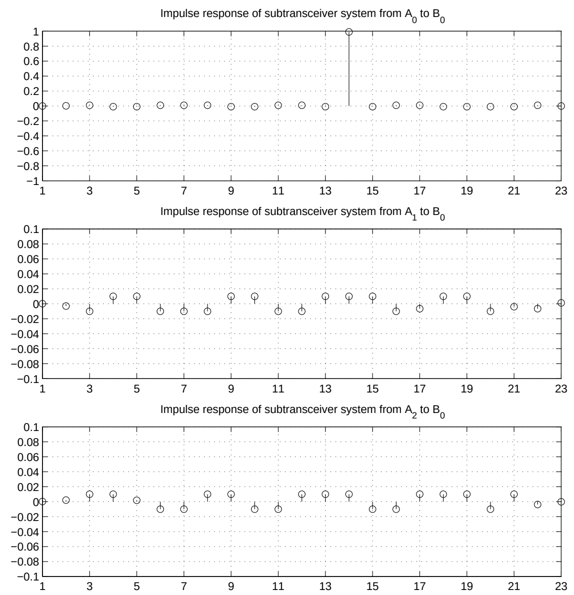

Figure 6: Impulse response of subtransceiver from point A_i to B_0 ($i = 0, 1, 2.$)

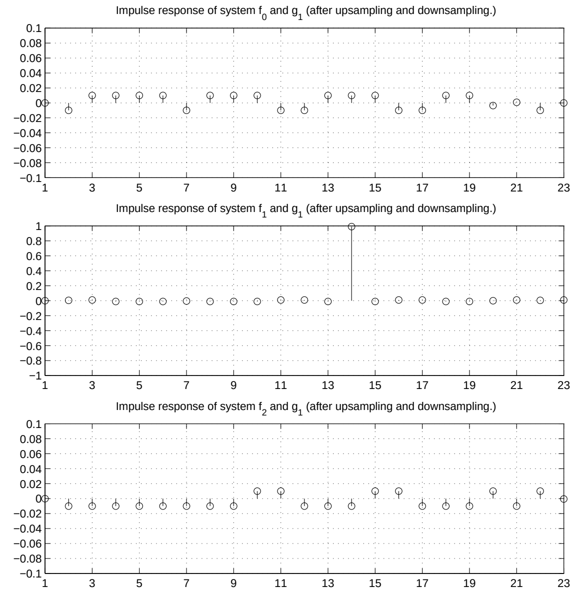

Figure 7: Impulse response of subtransceiver from point A_i to B_1 ($i = 0, 1, 2.$)

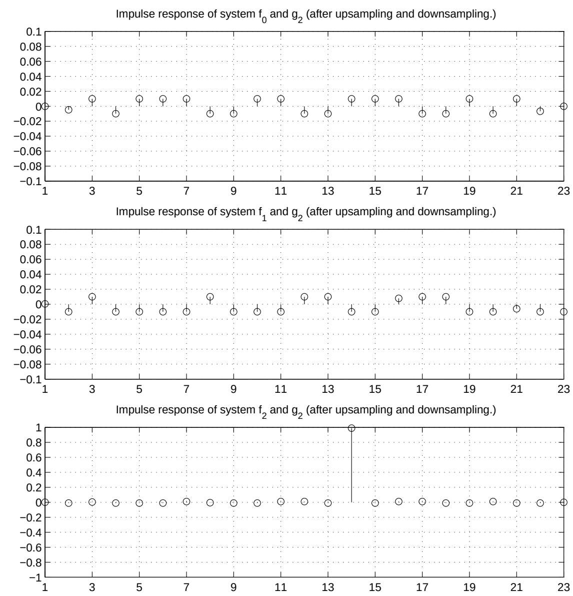

Figure 8: Impulse response of subtransceiver from point A_i to B_2 ($i = 0, 1, 2.$)