# Theoretical Performance of a BCH Hybrid-ARQ Protocol Operating in a Non-Perfect Reverse Channel

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## Abstract

**ARQ and Hybrid-ARQ protocols, which can provide an error-free digital communication link over wire-line or wireless channels, have received much attention in the recent years, [1], [2]. This letter presents a new approach for developing an efficient hybrid-ARQ protocol for links with no perfect reverse channel, using BCH codes. Attention is given for bounding the average time delay experienced by a single data frame, while keeping the throughput high. The average throughput and the time delay are compared to those of other recent protocols.**

**Index Terms-- Hybrid-ARQ Schemes, Non-Perfect Reverse Channels, BCH Codes, Digital Communication Links**

## 1. INTRODUCTION

Application of hybrid-ARQ schemes, particularly type II, to improve the error-free throughput in wireless channels, was first considered in [2]. Many other varieties, which used block codes, have been presented over the years, [3], [4], [5]. However, the new scheme described in this paper, increases the throughput by minimizing the amount of parity bits transmitted. In addition, unlike in all previous publications, the reverse channel is not assumed to be perfect but considered to be as noisy as the forward channel.

It will be shown that the new BCH hybrid-ARQ protocol, [6], provides better performance in fully loaded, noisy forward and reverse channels. The BCH codes can correct multiple number of errors ($t$) using parity words of different lengths ($n$-$k$), in codewords of a large range of lengths ($n$), where $k$ denotes the length of data[7].

With the new protocol, first, the data frame consisting of both data and control fields, is transmitted after appending error detection checksum generated by a cyclic redundancy code (CRC). This checksum is very short and usually only about 16 to 32-bits. The receiver will send back a *NAck_P* indicating that it needs error-correction parity bits if it detects errors in this data frame. The transmitter will then transmit only the parity frame generated by the BCH encoder, which is of much shorter length compared to the data frame.

The receiver then will combine the parity frame with the original data frame and performs the normal forward error correction. If the total number of errors in both frames is within the error correction capability of the code, the data frame will be corrected of errors. An *Ack* will be issued and the transmitter can send a new frame. Otherwise, the receiver can request either a copy of the same parity frame again or a copy of the original data frame by sending a *NAck_P* or a *NAck_D* respectively. In this paper, the latter option is considered where a copy of the data frame is retransmitted if the receiver fails to correct errors using only a single parity frame. In this way, a data frame or the corresponding parity frame is retransmitted alternatively until an error-free frame is obtained.

Because the reverse channel is noisy too, the acknowledgements are repeated in consecutive frames. This bounds the total time delay because in case the first acknowledgement is in error, those frames which arrive immediately after, contain the acknowledgement for the same data frame. If the noise level is so high that it needs to repeat more than a preset number of acknowledgements, the data transmission rate or the symbol rate may be changed, [8]. This action is supposed to keep the throughput efficiency and the normalized average time delay at the same level.

In this particular investigation, each acknowledgement is repeated in 3-consecutive frames and whenever the frame error rate and then the throughput falls to about 50%, the data rate is decreased. In this kind of a channel, therefore, one out of two acknowledgements should arrive safely. For additional safety, 3-acknowledgements are sent. On the other hand, theoretically, there is a slight probability that all 3-frames are still in error. Therefore, the introduction of timers is necessary upon expiration of which the transmitter will request a new acknowledgement. This will stop the protocol from becoming unstable due to lost acknowledgements. However, the probability of this event is ignored in the analysis. The frame architecture of this protocol is shown below.

Length of data frames, $L = k$+CRC = D+C1+CRC
Length of parity frames, $L_P = n\text{-}k$+C2 = P+C2
Length of an *Ack* frame, $L_A$ = C1+CRC
Starting delimeter, $S$ = 8-bits

where,

D = Data bits
C1 = 32-bits, 1-byte for control & sequence numbers and 3-bytes for multiple acknowledgements
C2 = 8-bits, 1-byte for control & sequence numbers.
CRC = 16-bit CRC parity bits

## 2. AVERAGE NORMALIZED THROUGHPUT

The average throughput of this BCH Hybrid-ARQ protocol is dependent on the lengths of the data, parity and acknowledgement frames and on the mean number of different types of frames transmitted in order to completely transfer one error-free data frame. To evaluate the maximum link throughput, fully loaded forward and reverse channels are considered. This allows the use of piggybacked acknowledgements at all times without sending separate acknowledgement frames. In this paper, only the formulae obtained for the long-term average throughput, $\gamma_{H\text{-}ARQ}$, and the time delay, $M_{tD}$, using theoretical analysis of the protocol are given.

Say, the Gaussian channel noise introduces random bit errors to the transmitted data and the probability of bit error is $p$. Then, the long-term average throughput, is found to be, [6],

$$\gamma_{H-ARQ} = \frac{D}{[(L+S)(1+m_{nrf}) + (L_P + S)m_{nrp}]} \quad (1)$$

where the mean number of data frames transmitted except the original, $m_{nrf}$, is

$$m_{nrf} = \frac{P_{rp}P_{rf}}{1 - P_{rp}P_{rf}} \quad (2)$$

and the mean number of parity frames transmitted, $m_{nrp}$, is

$$m_{nrp} = \frac{P_{rp}}{1 - P_{rp}P_{rf}} \quad (3)$$

Here, $P_{rp}$ and $P_{rf}$, as shown in equations (4) and (5) below, represent the probability that a parity frame is requested, a *NAck_P* is created, and the probability that a copy of the same data frame is requested, a *NAck_D* is created by the receiver.

$$P_{rp} = 1 - (1-p)^L \quad (4)$$

$$P_{rf} = P_{rp}\Big(1 - \sum_{x=1}^{t}\Big\{ {}^{(L-CRC)}C_x\,[p^x(1-p)^{(L-CRC-x)}] \times \sum_{y=0}^{t-x} {}^{P}C_y\,[p^y(1-p)^{(P-y)}] \Big\} \Big) \quad (5)$$

where $x$ and $y$ are the number of errors in the data and parity frame such that $(x + y) =< t$, the error correcting capability of the BCH code.

For comparison, a normal selective repeat ARQ scheme with the same C1 control field for bounding the time delay in imperfect reverse channels, is used. The throughput of such a scheme is then given by

$$\gamma_{ARQ} = [D(1-p)^L]/[L+S] \quad (6)$$

## 3. AVERAGE TIME DELAY

The time delay is a function of the one-way propagation delay, $\tau$, and the transmission times, $t_L$, and $t_P$, taken by data and parity frames respectively. In addition, at the transmitter, the average waiting time for receiving a correct acknowledgement and the average waiting time for completion of sending the previous frame, and at the receiver, the average waiting time for piggybacking, have been included in the analysis. However, the possibility that all 3-acknowledgements sent for a given received data frame is in error and the timers expire is considered negligible. The processing delays at both ends, too, are considered to be negligible compared to the rest.

The average time delay of the BCH Hybrid-ARQ protocol, $M_{tD}$, is presented normalized to the no-protocol delay, $\tau + t_L$, which is the time taken to transmit a single data frame if there were no link protocol. Again the proof is not shown here.

***Case 1*:** The propagation delay is very small, $\tau << t_L$

$$M_{tD} = 1 + \frac{(az+b)\left(3 + \frac{t_P}{t_L}\right)}{(1-z)^2} + \frac{4z(a+b)}{(1-z)^2} \quad (7)$$

where

$$a = (1-P_{rp}); \quad b = P_{rp}(1-P_{rf}); \quad z = P_{rp}P_{rf} \quad (8)$$

and $t_P/t_L = (L_P+S)/(L+S)$, is simply the ratio of parity and data frame lengths including the flag lengths for any data transmission rate.

***Case 2*:** The propagation delay, $\tau = jt_L$ where $j$ = 1,2,3…

$$M_{tD} = 1 + \frac{(az+b)\left(2j+3+\frac{t_P}{t_L}\right)}{(j+1)(1-z)^2} + \frac{z(a+b)(2j+4)}{(j+1)(1-z)^2} \quad (9)$$

***Case 3:*** The propagation delay is very large, $\tau >> t_L$

$$M_{tD} = 1 + \frac{2(az+b)}{(1-z)^2} + \frac{2z(a+b)}{(1-z)^2} \quad (10)$$

## 4. RESULTS

The normalized average throughput of a number of length 1023 BCH hybrid-ARQ schemes, is graphed in Fig. 1 against the bit energy to noise ratio, $E_b/N_0$, and in Fig. 2 against the channel bit error probability, $p$, assuming that the modulation scheme is binary phase shift keying (BPSK).

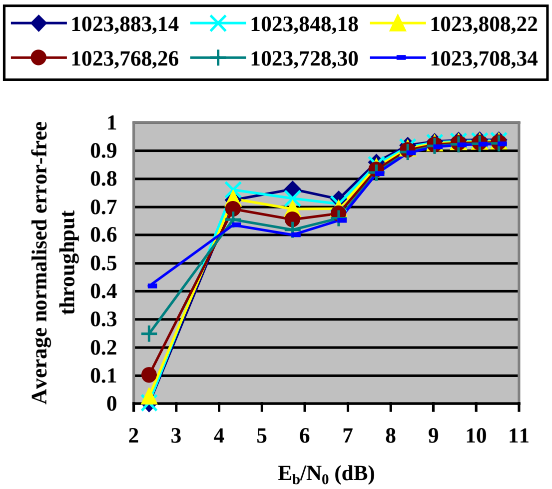


**Fig. 1: Average Normalized Throughput Graphed against the Bit Energy to Noise Ratio for BPSK**

The Fig. 3 compares the throughput of the new protocol with that of an ordinary selective-repeat ARQ protocol. Both protocols utilize a non-perfect reverse channel using the C1 control field. Results are presented for the BCH (1023, 883, 14) code where $D$ = 851, $P$ = 140, $L$ = 899 and $L_P$ = 148.

The performance improvement of the new BCH-hybrid ARQ protocol compared to other hybrid-ARQ which use block codes is evidenced by referring to [2], [3], [4], [5]. For example, it can be seen that the average normalized throughput of the original type II shown in [2] is about 0.5 and that of the new BCH hybrid-ARQ is 0.7 when $p = 10^{-2}$. The new protocol is superior until the error probability decreases to about $10^{-6}$. The throughput of the scheme in [5] is clearly inferior in the region where $p$ is between $10^{-2}$ and $10^{-3}$, obviously due to the excessive amount of parity transmissions. The scheme in [4] seems to be slightly superior with respect to the throughput but once the perfect reverse channel assumption is removed, it should become inferior.

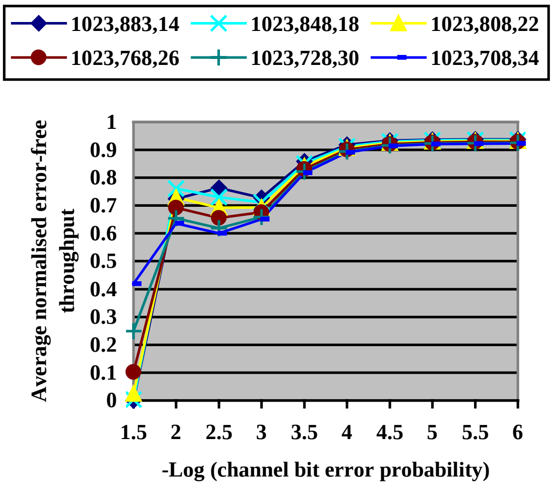


**Fig. 2: Average Normalized Throughput against the channel error probability for BPSK**

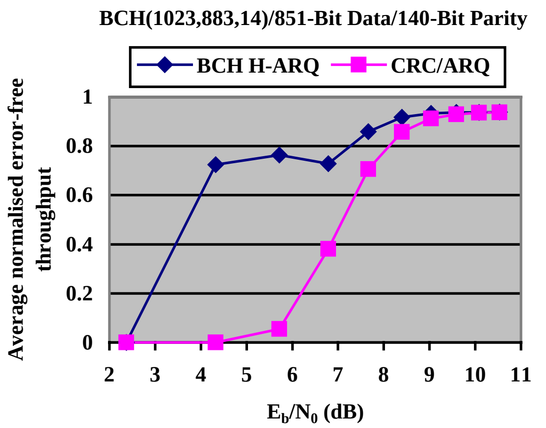


**Fig. 3: Comparison with a Normal ARQ Scheme**

The average time delay normalized to the no-protocol delay calculated using the equations (7) to (10) is illustrated in Fig. 4 for the BCH (1023, 883, 14) hybrid-ARQ protocol. This delay is expected to be shorter compared to those of others which complete the transmission of parity in parts, [3],[4], particularly when the channel error probability is high.

The Fig. 5 shows the probability of excessive time delay experienced by a single data frame. For example, the probability that the time delay is more than 6-times the no-protocol delay is almost zero.

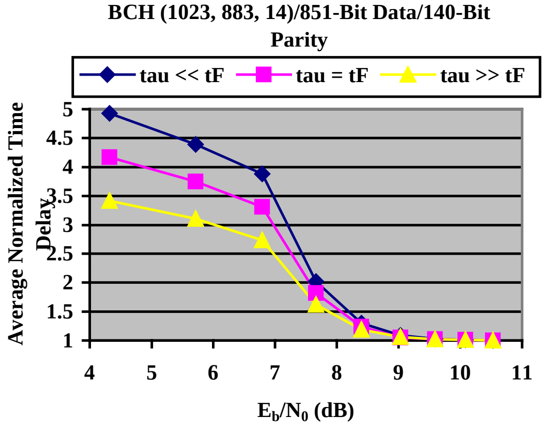


**Fig. 4: Average Time Delay Normalized to the No-Protocol Delay, $\tau$+$t_L$, for Different Propagation Delays, $\tau$**

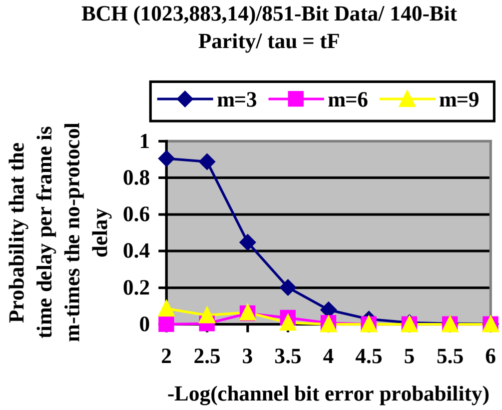


**Fig. 5: Probability of Excessive Time Delay**

# 5. CONCLUSIONS

It can be seen that the throughput advantage of the new protocol over not only the normal ARQ schemes but also the other type II hybrid-ARQ is considerable. This is evident particularly at the error probabilities between $10^{-2}$ and $10^{-4}$.

The average time delay can be high in non-perfect reverse channels. However, the delay can be bounded using multiple acknowledgements. It can be seen from the results, that the average time delay approaches the no-protocol delay when the channel error rate decreases to $10^{-4}$at an $E_b/N_0$ of roughly 8dB.

# 6. REFERENCES


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