Interleaved PC-OFDM to reduce the peak-to-average power ratio

A. D. S. Jayalath and C. Tellambura
School of Computer Science and Software Engineering
Monash University, Clayton, VIC, 3800.
e-mail:jayalath@csse.monash.edu.au

Abstract— **Parallel combinatory orthogonal frequency division multiplexing (PC-OFDM) yields lower maximum peak-to-average power ratio (PAR), higher bandwidth efficiency and lower bit error rate probability (BER) on Gaussian channels compared to OFDM systems. However, PC-OFDM does not improve the statistics of PAR significantly. In this paper, the use of a set of fixed permutations to improve the statistics of the PAR of a PC-OFDM signal is presented. For this technique, $K-1$ interleavers are used to produce $K-1$ permuted sequences from the same information sequence before coding using PC-OFDM. The Sequence with the lowest PAR, among K sequences is chosen for the transmission . The PAR of a PC-OFDM signal can be further reduced by 3-4 dB by this technique. Mathematical expressions for the complementary cumulative density function (CCDF) of PAR of PC-OFDM signal and interleaved PC-OFDM signal are also presented.**

Key Words:OFDM, Peak-to-average power ratio, PC-OFDM, Data permutation.

I. INTRODUCTION

Multicarrier modulation such as OFDM has been investigated and demonstrated as an asymptotically optimal technique for high speed data transmission. OFDM systems are also robust to severe multipath fading [1]. Moreover, OFDM transceiver can be implemented efficiently using the fast Fourier transform algorithm. OFDM applications include digital audio broadcasting, asynchronous digital subscriber lines, digital video broadcasting and wireless local area networks. Due to the nonlinearities of transmitter power amplifiers, high PAR values of the OFDM signal generate out-of-band noise (OBN). There are restrictions imposed by the Federal communications commission and other regulatory bodies on the level of these spurious transmissions. These restrictions impose a maximum output power limitation. This output power limitation corresponds to what is known as power amplifier back-off. OFDM usually needs higher power amplifier back-offs to meet these requirements. Further more nonlinearities of amplifier cause inband distortions of the signal giving higher BERs. Back-off required at the amplifier can be reduced by designing OFDM systems with low PAR.

Solutions to the PAR problem of OFDM have been proposed by several researchers [2,3]. PC-OFDM [4] is such a scheme proposed to reduce the upper bound of the PAR. PC-OFDM is generated by inserting zero amplitude subcarriers into an OFDM signal. To increase the system throughput a set of bits called PC bits are mapped to zero amplitude positions. PC-OFDM can carry more information bits than a similar OFDM system depending on the number of zero amplitude subcarriers. PC-OFDM expands the signal constellation by adding a zero amplitude signal point. This reduces the number of active subcarriers of an OFDM signal thus reducing the PAR. This reduction in PAR is measured in terms of the theoretically maximum PAR, $10\log_{10}(N)$, where N is the number of active carriers. Despite increasing the number of zero subcarriers, PC-OFDM offers negligible improvements in the statistics of the PAR. However, statistical properties of PC-OFDM can be improved further by using other PAR reduction methods.

In this paper, the performance of interleaved PC-OFDM in improving PAR statistics is investigated. Interleaved PC-OFDM is obtained by combining the data permutation technique reported in [5] with PC-OFDM. Large PAR reductions can be achieved by interleaved OFDM at the expense of system complexity. The PAR statistics of an OFDM signal is evaluated by using the complementary cumulative distribution function $(\text{CCDF} = \Pr[\text{PAR} > \text{PAR}_0])$. Mathematical expressions to evaluate the CCDF of PC-OFDM signal and interleaved PC-OFDM signal are also presented.

In Section 2 the OFDM system model and the theoretical description of the system are given. Section 3 presents the non-linear transmitter power amplifier characteristics. The simulation and theoretical results are presented in Section 4. Section 5 concludes the paper.

II. SYSTEM MODEL

A. An OFDM system PAR

A block of N symbols, $X_n, n = 0, 1, ..., N-1$, is formed with each symbol modulating one of a set of N subcarriers with frequency, $f_n, n = 0, 1, ..., N-1$. The N subcarriers are chosen to be orthogonal, that is $f_n = n\Delta f = n/T$, where and T is the OFDM symbol duration. The complex baseband signal can be expressed as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi f_n t}, 0 \leq t \leq T. \quad (1)$$

This signal can be generated by taking an N point inverse discrete Fourier transform (IDFT) of the block X_n followed by low pass filtering. The actual transmitted signal is modelled as real$\left[x(t)e^{j2\pi f_c t}\right]$, where f_c is the carrier frequency. The PAR of the transmitted signal in (1) can be defined as

$$\xi = \frac{\max |x(t)|^2}{E[|x(t)|^2]} \quad (2)$$

where $E[x]$ is the expected value of x. The PAR of the continuous-time OFDM signal cannot be computed precisely by the use of the Nyquist sampling rate [6], which amounts to N samples per symbol. In this case, signal peaks are missed and PAR reduction estimates are unduly optimistic. Oversampling by a factor of 4 is sufficiently accurate. and is achieved by the simple computation of the $4N$-point IDFT of the data frame. Upper bound of the PAR for an N subcarriers OFDM system is N. Therefore we can reduce the upper bound by setting some subcarriers to zero. This is what motivates PC-OFDM.

B. PC-OFDM

Parallel combinatory OFDM signals reported in [4] can be described as follows. For $N = N_{null} + N_{pc}$, consider N_{pc} subcarriers with M-PSK signal points and N_{null} subcarriers with zero amplitude points. This gives a system with the maximum PAR of $10\log_{10}(N_{pc})$ dB, which is lower than the maximum PAR of an OFDM system with N subcarriers. This new system is called a parallel combinatory OFDM system. In the transmitter we first choose which subcarriers to be zero and nonzero respectively. Then N_{pc} subcarriers are modulated by M-phase shift keying (M-PSK) constellation points. Thus m_{psk}, $(N_{pc} \times \log_2(M))$ bits are mapped to a subset of subcarriers used. Choosing N_{pc} out of N subcarriers can be done in $\binom{N}{N_{pc}}$ different ways. The total number of bits (m_{tot}) that can be transmitted using a PC-OFDM signal can be found as

$$m_{tot} = m_{psk} + m_{pc} = N_{pc}\log_2 M + \left\lfloor \log_2 \binom{N}{N_{pc}} \right\rfloor \quad (3)$$

where $\lfloor x \rfloor$ is the largest integer smaller than, or equal to x.

Reference [4] suggests to use a maximum likelihood (ML) detector at the receiver after the FFT operation to recover the data. The ML-detector maps the signal points in the subcarriers to the closest signal point in the $(M+1)$-APSK (amplitude and phase shift keying) constellation. If exactly N_{pc} subcarriers are mapped to signal points with non-zero amplitudes, the received symbol is accepted and decoded. Otherwise, if there are more subcarriers with zero amplitude, $N_{null}+L$, L subcarriers close to any of the nonzero constellation points are decoded to the corresponding non zero point in the $(M+1)$-APSK constellation. If the number of zero subcarriers is less than N_{null}, $(N_{null} - L)$, L nonzero subcarriers having the smallest amplitudes are set to zero. Received bits are decoded after these corrections.

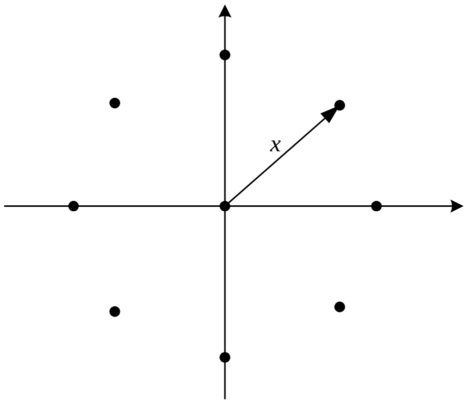

Fig. 1. Signal constellation for 9-APSK.

C. $(M+1)$-APSK Signal constellation

The $(M+1)$-APSK signal constellation is constructed from an M-PSK constellation. There is a signal point at the origin of the signal space. The amplitudes of the other M constellation points depend on the N/N_{pc} ratio. It can be expressed as

$$x = \sqrt{\frac{N}{N_{pc}}} \quad (4)$$

Therefore, for each PC-OFDM system we have to define a unique signal constellation. Figure 4 shows the 9-APSK signal constellation.

The bandwidth efficiency of PC-OFDM may be higher than OFDM. The problem of mapping bits to PC-OFDM signal is solved by using the Jonson association scheme, together with a position algorithm for the PSK symbols [4].

D. Interleaver approach

The interleaver approach to reduce PAR of an OFDM signal is presented in [5]. In this approach $K-1$ interleavers are used at the transmitter. These interleavers produce $K-1$ permuted frames of the input data before mapping symbols. The four times oversampled IDFT of each frame (including the original frame) is used to compute its PAR. The minimum PAR frame of all the K frames is selected for transmission. The identity of the corresponding interleaver is also sent to the receiver as side information. A similar approach has been independently developed by Ochiai and Imai [7].

E. *Theoretical CCDF of PAR of PC-OFDM*

Reference [8] provides an empirical formula for the distribution of the PAR. Reference [9] derives an expression for the exact distribution of the PAR. As this expression is complex and difficult to solve an approximation of the peak distribution based on level-crossing rate is also presented. This simple expression is used here to predict the behavior of the interleaved PC-OFDM signals.

The CCDF of PAR of an OFDM signal can be expressed as [9]

$$\Pr(\xi > \xi_0) \approx \Pr(\xi > \overline{\xi}|\xi_0 > \overline{\xi}) = \begin{cases} 1 - \left(1 - \frac{\xi_0 e^{-\xi_0{}^2}}{\overline{\xi} e^{-\overline{\xi}^2}}\right)^{\sqrt{\frac{\pi}{3}} N \overline{\xi} e^{-\overline{\xi}^2}} & \text{for } \xi_0 > \overline{\xi}, \\ 1 & \text{for } \xi_0 \leq \overline{\xi}. \end{cases} \quad (5)$$

This was derived considering only the peaks exceeding a given threshold $\overline{\xi}$ above zero. A proper $\overline{\xi}$ is selected by making the assumption: *each positive crossing of the level $\overline{\xi}$ has a single positive peak that is above the level $\overline{\xi}$* [9]. It also suggests $\overline{\xi} = \sqrt{\pi}$ for QPSK modulation and slightly lower value for 16-QAM. Next, we derive an expression for the PAR distribution of PC-OFDM signals.

The PC-OFDM signal has fixed number of zero amplitude subcarriers. Based on (5) we can express the CCDF of PAR of the PC-OFDM signal as follows:

$$\Pr(\xi > \xi_0) = \begin{cases} 1 - \left(1 - \frac{\xi_0 e^{-\xi_0{}^2}}{\overline{\xi} e^{-\overline{\xi}^2}}\right)^{\sqrt{\frac{\pi}{3}} N_{pc} \overline{\xi} e^{-\overline{\xi}^2}} & \text{for } \xi_0 > \overline{\xi}, \\ 1 & \text{for } \xi_0 \leq \overline{\xi}. \end{cases} \quad (6)$$

Now, for the interleaved PC-OFDM signal we assume that all K sequences are independent and uncorrelated. The validity of the independence assumption is confirmed via simulations. Then the CCDF of PAR of the interleaved PC-OFDM signal can be expressed as

$$\Pr(\xi > \xi_0) = \begin{cases} \left[1 - \left(1 - \frac{\xi_0 e^{-\xi_0{}^2}}{\overline{\xi} e^{-\overline{\xi}^2}}\right)^{\sqrt{\frac{\pi}{3}} N_{pc} \overline{\xi} e^{-\overline{\xi}^2}}\right]^K & \text{for } \xi_0 > \overline{\xi}, \\ 1 & \text{for } \xi_0 \leq \overline{\xi}. \end{cases} \quad (7)$$

III. NONLINEAR TRANSMITTER CHARACTERISTICS

The nonlinear distortion at the transmitter causes interferences both inside and outside the signal bandwidth. The inside component determines the amount of bit error rate degradation of the system, whereas the outside component effects the adjacent frequency bands. In other words outside component increases the out of band radiation of the signal. The transmitter nonlinear distortion include signal clipping in the analog to digital (A/D) converter, signal clipping in the IFFT and FFT processors with a limited word length, amplitude modulation (AM)/AM and AM/phase modulation (PM) distortion in the radio frequency (RF) amplifiers. The OBN of OFDM signals increases due to nonlinear power amplifiers operating at lower back-offs. The high PAR of OFDM requires high back-offs at the amplifiers. The non-linear characteristics of the soft limiter is shown below [10]. The performance of PC-OFDM when passing through a soft limiter nonlinear device is simulated in this paper.

A. *Soft limiter (SL)*

Since the AM/PM component is zero the nonlinear characteristics of a SL can be written as

$$g(x) = \begin{cases} x & |x| \leq A \\ Ae^{j\varphi} & |x| > A \end{cases} \quad (8)$$

where x is the input to the SL, A is the saturated output and φ is the phase angle of the input x. Although most physical components will not exhibit this piecewise linear behavior, the SL can be a good model if the nonlinear element is linearized by a suitable predistortor. The back off (BO) at the non-linear device can be defined in terms of maximum power output A^2 as

$$\text{BO} = 10 \log_{10} \left\{ \frac{A^2}{E\{|x|^2\}} \right\} \quad (9)$$

where $E\{|x|^2\}$ is the average of the input power to the non-linear device. For PSD results, it is convenient to define the normalized bandwidth $B_n = fT/N$, where T is the OFDM symbol duration.

IV. RESULTS

PC-OFDM systems with $N = 32$, $N_{pc} = 12, 16, 20, 28$ and 5-APSK modulation are simulated. Figure 2 shows a comparison of theoretical and simulated CCDFs of PC-OFDM signals. The theoretical expression (6) approximates the simulation results well. Very little improvement of PAR statistics is observed when the number of zero subcarriers (N_{null}) in a PC-OFDM system is increased. Figure 3 depicts the CCDF of PAR of the PC-OFDM and interleaved PC-OFDM signals with $N_{pc} = 28$ and 12. No PAR reduction can be observed in PC-OFDM, when $N_{pc} = 28$ and there is about 0.5dB reduction in PAR at 0.1%PAR when $N_{pc} = 12$ (throughout the paper we denote by n%PAR, a value that PAR exceeds this value less than n% of

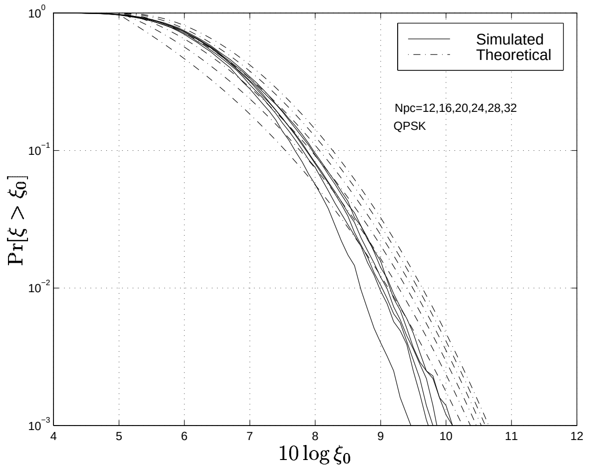

Fig. 2. Theoretical and Simulated CCDF of PC-OFDM.

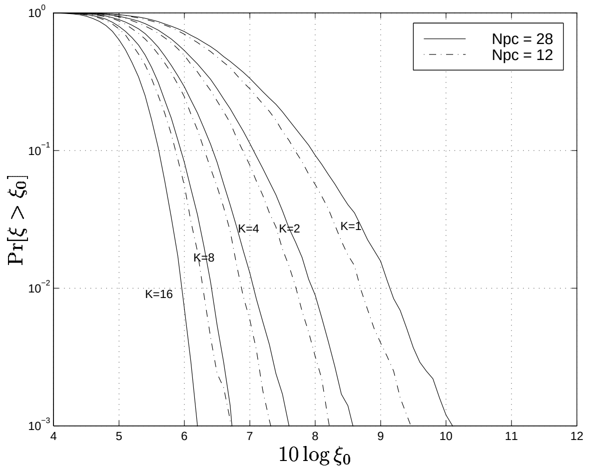

Fig. 3. CCDF of PC-OFDM with interleaving to reduce PAR (M=4).

OFDM blocks) compared to the PAR statistics of an OFDM system with $N = 32$ subcarriers.

Statistics of PAR is improved very slightly when the number of subcarriers in an OFDM system is reduced, although it gives a low value for the maximum possible PAR. For example, when $N = 32$ the maximum possible PAR is $10 \log_{10}(32)$ =15.1dB, while this is 14.4dB when $N = 28$ although there is a 1dB reduction in the maximum value of PAR, the statistics remain unchanged above the region 0.1%PAR. For $N = 32$ and 12, the difference in maximum PAR is about 4.3dB, while the statistics show only 0.5dB reduction at 0.1%PAR. Use of interleaving will not improve the maximum PAR but will improve the PAR statistics. Total PAR reduction of 4dB is observed for $N_{pc} = 28$ when interleaving (number of interleavers are 15, $K = 16$) is used. A 1.5dB improvement of PAR statistics at 0.1%PAR is observed even with a single interleaver. Similar performance is observed for PC-OFDM with $N_{pc} = 12$.

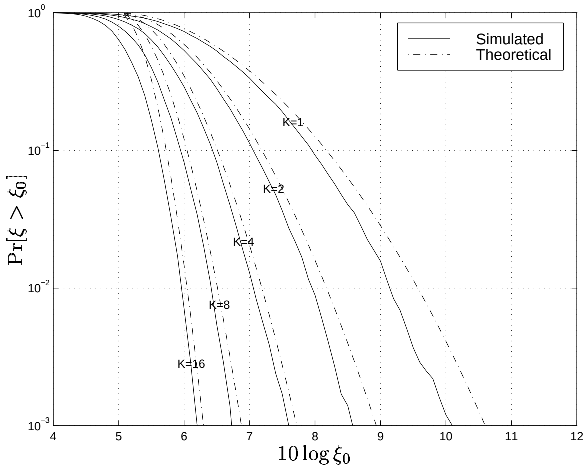

Fig. 4. Theoretical and Simulated CCDF of interleaved PC-OFDM $N_{pc} = 28$.

Figure 4 and Figure 5 compare theoretical and simulated CCDFs of interleaved PC-OFDM for $N_{pc} = 28$ and 20. Theoretical expression (7) can therefore predict the PAR statics of any interleaved PC-OFDM system. The complexity of a PC-OFDM system increases with the number of subcarriers. Therefore, the expressions (6) and (7) can predict the performance without performing extensive computer simulations.

Figure 6 shows the power spectral density of the PC-OFDM signal when passing through a SL non-linearity. A clear reduction in out of band ration is observed in interleaved PC-OFDM systems. Simulation results are shown for an PC-OFDM system with $N_{pc} = 28$ and 5-APSK modulation. When the clipping level of SL is at 5dB a 10dB reduction in out of band ration is observed at $2B_n$, while the reduction is more than 50dB when the clipping level is 7dB. To achieve similar performances with PC-OFDM SL clipping level has to be increased to 11dB. Therefore, interleaved PC-OFDM reduces the back-off by 4dB.

The use of interleaving does not have any effect on the bit error rate performance of PC-OFDM systems. Interleaved PC-OFDM systems therefore possess the superior performance in AWGN channels compared to OFDM systems.

V. CONCLUSIONS

This paper presents the use of data permutation in PC-OFDM to design systems with improved PAR statistics. PAR and BER is lower for PC-OFDM systems for high E_b/N_0. Use of interleaving will not effect the BER performance but reduces the PAR significantly. Theoretical expressions are presented to evaluate the CCDF of PC-OFDM and interleaved PC-OFDM signals. These formulae give simple means to evaluate the performance of different PC-OFDM systems with vary-

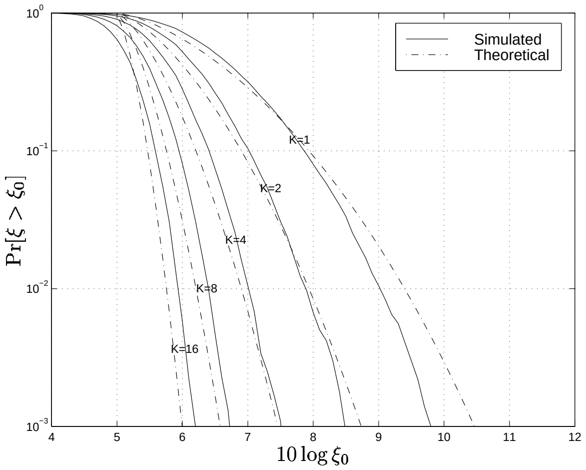

Fig. 5. Theoretical and Simulated CCDF of interleaved PC-OFDM $N_{pc} = 20$.

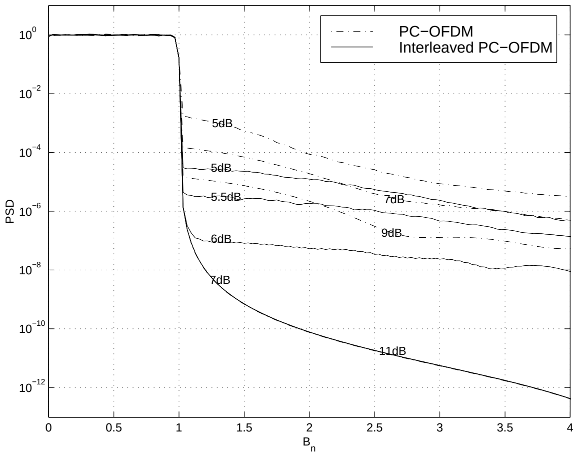

Fig. 6. Power spectral density of PC-OFDM and interleaved PC-OFDM.

ing number of nonzero subcarriers, without performing extensive computer simulations.

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