Novel Adaptive Modulation Scheme to Reduce both PAR and ICI of an OFDM Signal

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Abstract— **Orthogonal Frequency Division Multiplexing (OFDM), a promising technique to support high-bit-rate wireless systems, is limited by potentially large Peak-to-Average Power ratio (PAR) and Intercarrier Interference (ICI) due to frequency offset errors. Several techniques proposed in the literature treat these limitations as two separate problems. This paper shows that a simple adaptive modulation scheme is effective in reducing the both PAR and ICI simultaneously. We propose the use of M-point Zero-padded Phase Shift Keying (M-ZPSK), which includes a signal point of zero amplitude, as a modulation scheme. We evaluate the performance of the proposed scheme analytically and by simulation.**

Keywords— **Orthogonal Frequency Division Multiplexing, Peak-to-Average Power ratio (PAR), Carrier Frequency Offset (CFO), Intercarrier Interference (ICI)**

I. Introduction

Managing the explosive growth in mobile users and the demand for new services, such as wireless multimedia services and wireless Internet access, requires high-data-rate wireless communications systems. A promising transmission technique for the support of such large data rates, which has sufficient robustness to handle radio channel impairments, is OFDM. It has been accepted for several wireless LAN standards, as well as a number of mobile multimedia applications such as digital audio broadcasting (DAB), asynchronous digital subscriber lines (ADSL), digital video broadcasting (DVB) and IEEE 802.11a wireless LAN [1]. However, two major drawbacks of OFDM are its potentially large Peak-to-Average Power ratio (PAR) and Intercarrier Interference (ICI) due to frequency offset errors.

Due to the large number of subcarriers, the OFDM signal has a large dynamic signal range with high PAR. These high signal peak can lead to in-band distortion and spectral spreading in the presence of non-linear devices. This results in performance degradation and/or inefficiency in use of the non-linear devices. Increased interest in OFDM for many wireless applications and severe effects from PAR have motivated the search for PAR reduction techniques. In the open literature, several alternative techniques have been proposed to reduce PAR [2–9].

Another drawback of OFDM is ICI, which is caused by misalignment in carrier frequencies between the transmitter and receiver or by Doppler shift [10]. This can be a significant problem where low-cost mobile hand sets that cannot employ very accurate frequency estimators because of the cost involved. Several techniques have been proposed for reducing the ICI. These include frequency-domain equalization, time-domain windowing, self-ICI cancellation or polynomial coded cancellation, correlative coding and forward error correction codes [10–12].

The key idea for the PAR reduction technique in [8, 9] is to expand the M-PSK constellation with an extra, zero amplitude point. Thus, some subcarriers have zero amplitude. This reduces the peak of the OFDM signal. However, these subcarriers are not fixed and they should be selected by considering the entire OFDM block in such a way to reduce PAR. In [8], a bit mapping procedure using Johnson association is proposed to select those zero-amplitude subcarriers. In [9], the signal space is expanded by varying the non-modulated subcarrier set and the symbol pattern with low PAR is selected for transmission. This approach is suitable for small N as the transmitter and receiver need to maintain a lookup table. Interestingly, these two approaches offer higher bandwidth efficiency than that of a normal OFDM system with increased complexity as non-modulated subcarrier positions are also used to carry information bits.

Motivated by these two successful approaches [8, 9], we propose an M-ZPSK modulation scheme, which includes a signal point of zero amplitude. Thus, some subcarriers have zero amplitude. Note that both M-ZPSK and M-PSK have the same number of signal points. We now propose a simple adaptive M-ZPSK modulation scheme to reduce the both PAR and ICI simultaneously. Note that other techniques proposed in the literature treat the PAR and ICI problems as two separate issues. To the best of our knowledge, a technique that limits PAR and ICI together has not been reported before.

The organization of this paper is as follows: Section II presents the PAR and ICI problem in OFDM systems. We explain the adaptive M-ZPSK modulation scheme to reduce PAR in Section III. ICI reduction capability of the proposed scheme is presented in Section IV and the performance of the proposed scheme in the presence of non-linearities is explained in Section V. Concluding remarks are presented in Section VI.

II. OFDM Issues

A. PAR Problem

The complex baseband OFDM signal may be represented as [2, 3]

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} c_k e^{j2\pi k \Delta f t},\ 0 \le t \le T. \tag{1}$$

where $j^2 = -1$ and N is the total number of subcarriers. The frequency separation between any two adjacent subcarriers is $\Delta f = 1/T$ and T is the OFDM symbol duration. c_k is the data symbol for the k-th subcarrier. Each modulated symbol c_k is chosen from the set $F_M = \{\lambda_1, \lambda_2, .., \lambda_M\}$ of M distinct elements. The set F_M is called the *signal constellation* of the M-ary modulation scheme. We shall refer to $\mathbf{c} = (c_0, c_1, ..., c_{N-1})$ as a data frame or codeword, as appropriate and is a constellation symbols from a encoder.

The PAR of the transmitted signal in (1) can be defined as

$$\text{PAR}(\mathbf{c}) = \frac{\max |s(t)|^2}{E[|x(t)|^2]} \quad (2)$$

where $E[\cdot]$ denotes the average. Therefore, the maximum PAR of a baseband signal can be expressed as

$$\max \text{PAR}(\mathbf{c}) = N. \quad (3)$$

To more accurately approximate the true PAR, $s(t)$ has to be oversampled. In this study, we consider oversampling by a factor of four as further increase in oversampling does not influence the PAR. The complementary cumulative density function (CCDF) of the PAR of an OFDM signal can be expressed as [3,6]

$$\Pr\{\text{PAR} > \text{PAR}_0\} = 1 - (1 - \exp\{-\text{PAR}_0\})^{\alpha N} \quad (4)$$

where $\alpha = 1$ for non-oversampling case and $\alpha = 2.8$ for oversampling case.

B. ICI Problem

We assume that $s(t)$ is transmitted on an additive white Gaussian noise channel, and so the received signal sample for the k-th subcarrier after Fast Fourier Transform(FFT) demodulation can be written as [10]

$$y_k = c_k S_0 + \sum_{l=0, l\neq k}^{N-1} S_{l-k} c_l + n_k \,;\, k = 0, ..., N-1 \quad (5)$$

where n_k is a complex Gaussian noise sample (with its real and imaginary components being independent and identically distributed with variance σ^2) and c_k is one of the signal constellations. The second term in (5) is the ICI term attributable to the CFO. The sequence S_k (the ICI coefficients) depends on the CFO and is given by [10]

$$S_k = \frac{\sin \pi(k+\varepsilon)}{N \sin \frac{\pi}{N}(k+\varepsilon)} \exp[j\pi(1-\frac{1}{N})(k+\varepsilon)] \quad (6)$$

where ε is the normalized frequency offset defined as a ratio between the frequency offset (which remains constant over each symbol period) and the subcarrier spacing. For a zero frequency offset, S_k reduces to the unit impulse sequence. The ICI term can be expressed as (5)

$$I_k = \sum_{l=0, l\neq k}^{N-1} S_{l-k} c_l, \text{ for } 0 \leq k \leq N-1. \quad (7)$$

Note that I_k is a function of both c_k and ε.

III. PAR Reduction by Adaptive Modulation

A. M-ZPSK Modulation Scheme

The signal constellation of M-ZPSK can be expressed as

$$F_M = [0, \alpha, \alpha\xi, ..., \alpha\xi^{M-2}] \quad (8)$$

where $\alpha = \sqrt{\frac{M}{M-1}}$ and $\xi = e^{j\frac{2\pi}{M-1}}$. The average power of these signal points is one. Further, this new mapping does not require any redundant subcarriers to deliberately introduce zero amplitude symbol in an OFDM block. Thus, the bandwidth efficiency and the complexity of this mapping is the same as normal OFDM system, unlike in [8, 9].

However, one disadvantage of M-ZPSK over conventional M-PSK is the degradation in signal-to-noise ratio (SNR) as signal points are little closer in M-ZPSK. The minimum Euclidean distance of M-ZPSK is $\min\{2\sqrt{\frac{M}{M-1}}\sin\{\frac{\pi}{M-1}\}, \sqrt{\frac{M}{M-1}}\}$ whereas that of M-PSK is $2\sin\{\frac{\pi}{M}\}$. For an example, the minimum Euclidean distance is 1.4142 for 4-PSK and 1.1547 for 4-ZPSK.

B. Adaptive Approach for PAR Reduction

The maximum PAR of a baseband OFDM signal with M-ZPSK is expressed as (3)

$$\max \text{PAR}(\mathbf{c}) = N - L \quad (9)$$

where L is the number of zero amplitude signal points in the OFDM block. Note that L is random and it determines the PAR reduction capability of the M-ZPSK mapping. Thus, PAR reduction may not be significant for small L.

If all the signal points in M-ZPSK have the same a priori probability, then, L is equal to $\frac{N}{M}$. For $N = 128$ and M=4, the maximum PAR is 19.8 dB whereas that of conventional MPSK is 21 dB. However, the PAR reduction of 1.2 dB due to M-ZPSK mapping may not be realized as maximum PAR occurs very rarely in practice.

To exploit the characteristics of the M-ZPSK mapping, we propose an adaptive scheme to reduce PAR for any random data. In this scheme, one out of M bit patterns of $\log_2 M$ bits can be mapped to a signal constellation of zero amplitude. This leads to M independent OFDM frames representing the same information. Therefore, the OFDM frame with low PAR (say PAR_{low}) can be selected for transmission. The block diagram for this scheme is shown in Fig. 1.

The CCDF of the PAR of this scheme can be expressed as [3,6]

$$\Pr\{\text{PAR} > \text{PAR}_{low}\} = \{1 - (1 - \exp\{-\text{PAR}_0\})^{\alpha N}\}^M. \quad (10)$$

To recover the data, the receiver has to know which mapping has actually been used in the transmitter. This can be achieved by transmitting $\log_2 M$ bits as Side Information (SI). As these bits are most important, they should be protected by channel coding. The proposed scheme requires M number of data mapping blocks and IFFT blocks in the transmitter. Moreover, this approach applies to an arbitrary number of subcarriers. Note that this scheme does not require any multiplication and complex optimization as in [6]. Mapping of bits to constellation symbols is a rather simple operation comparing with multiplication.

The reduction capability of the proposed scheme is restricted by modulation scheme as M determines the number of independent OFDM frames representing the same information. Yet, this is not an issue as high peaks occur infrequently and it is desirable to keep the complexity minimum. Another important advantage

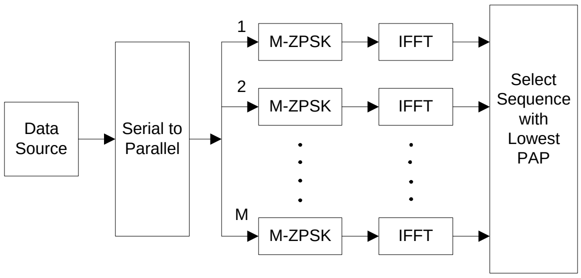

Fig. 1. Block Diagram for Adaptive OFDM Scheme

of this scheme is reduced sensitivity to frequency offset errors. We will elaborate this in the sequel.

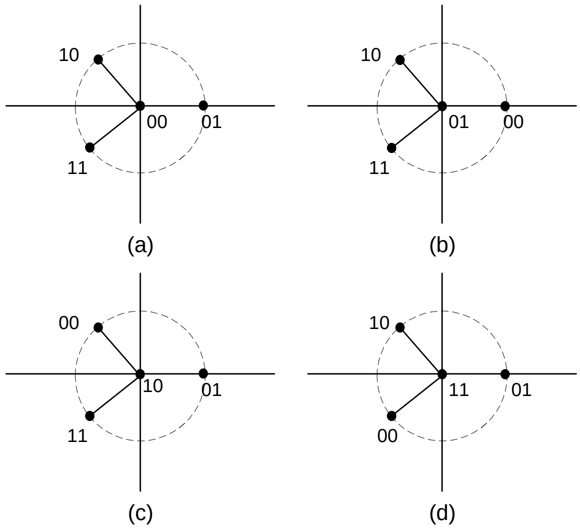

Fig. 2. Mapping Signal Constellations for Adaptive OFDM System for $M = 4$

Adaptive 4-ZPSK signal constellation mapping is shown in Fig. 2. The input bit sequence is mapped in four different ways and the mapped symbols are passed through IFFT processor. The PAR of the resulting four sequences are calculated by oversampling. The lowest PAR sequence is selected for transmission. Fig. 3 shows the CCDF of the PAR of an OFDM signal with $N = 128$. The PAR of an adaptive 4-ZPSK scheme exceeds 9.5 dB for only 1 out of 10^4 whereas that of conventional scheme is 1 out of 10. Further, simulation results are also confirmed by theoretical expressions in (4) and (10).

C. BER Analysis

We use an analytical method due to Craig [13] to derive the BER. This method is applicable to any decision region of polygon as shape. Zero amplitude symbol, C_0, has a decision boundary of a circle with radius a where a is equal to $\frac{1}{2}\sqrt{\frac{M}{M-1}}$. The other $(M-1)$ signal points $(C_1, C_2, ..., C_{M-1})$ are evenly placed on a circle with radius $\sqrt{\frac{M}{M-1}}$. Therefore, the decision region is same for those $(M-1)$ signal points. Thus, the probability of symbol error given C_n transmitted, $\Pr\{\text{error}/C_n\}$, is equal for $n = 1, 2, ..., M-1$.

To make the boundaries of the decision region as a polygon, we first approximate the arc boundaries of the decision region

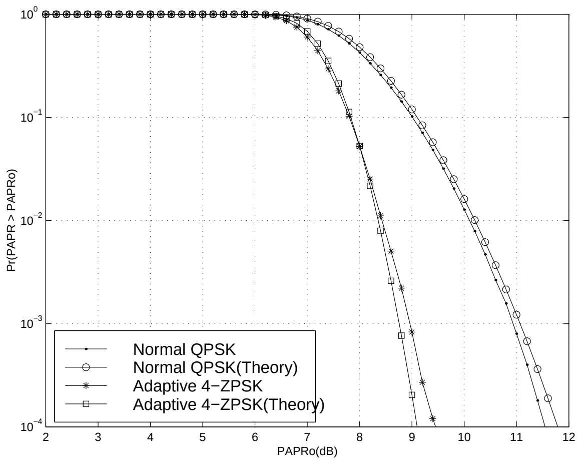

Fig. 3. CCDF of the PAR of an OFDM Signal for $M = 4$.

as straight lines. Thus, the decision region of the symbol C_0 becomes a polygon with $2(M-1)$ sides of equal length. Next, we approximate the infinite decision region of symbol C_1 as a polygon with six sides. This approximation can be justified if the shortest distance (say b) from the symbol C_1 to the side on the infinite region is large enough. Now, we can use the approach in [13] to calculate the probability of symbol error. Fig. 4 shows the constructed boundaries of the decision region as a polygon.

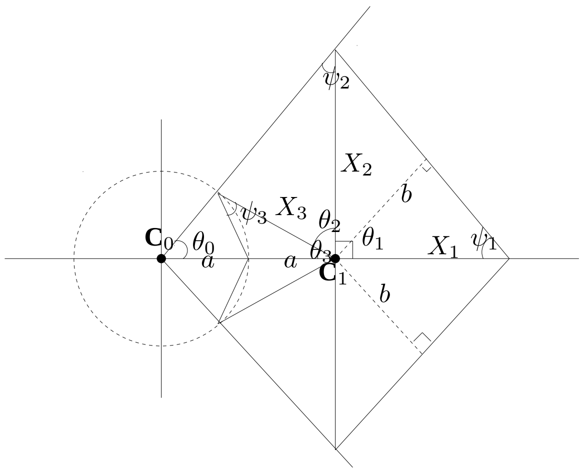

Fig. 4. Typical M-ZPSK Signal Constellation and Approximate Decision Boundaries

The probability of symbol error given C_1 transmitted is expressed as [13]

$$\Pr\{\text{error}/C_1\} = \frac{2}{\pi}\sum_{k=1}^{3}\int_0^{\theta_k} \exp\{-\frac{X_k^2 \sin^2(\psi_k)}{2\sigma^2 \sin^2(\theta+\psi_k)}\}d\theta. \tag{11}$$

Note that the corresponding angles and lengths are marked on Fig. 4 and can be calculated by trigonometric manipulations. The length b can be selected arbitrarily large enough (We use $b = 3a$).

The probability of symbol error given C_0 transmitted is expressed as [13]

$$\Pr\{\text{error}/C_0\} = \frac{2(M-1)}{\pi}\int_0^{\theta_0} \exp\{-\frac{a^2 \sin^2(\psi_0)}{2\sigma^2 \sin^2(\theta+\psi_0)}\}d\theta. \tag{12}$$

Assuming equal a priori probability for all signal points, the probability of symbol error can be expresses as

$$P_S = \frac{1}{M}\left[\Pr\{\text{error}/C_0\} + (M-1)\Pr\{\text{error}/C_1\}\right]. \tag{13}$$

Then, the average probability of bit error is expressed as [14]

$$P_b = \frac{1}{\log_2 M} P_S. \tag{14}$$

Therefore, (14) can be evaluated numerically for M-ZPSK.

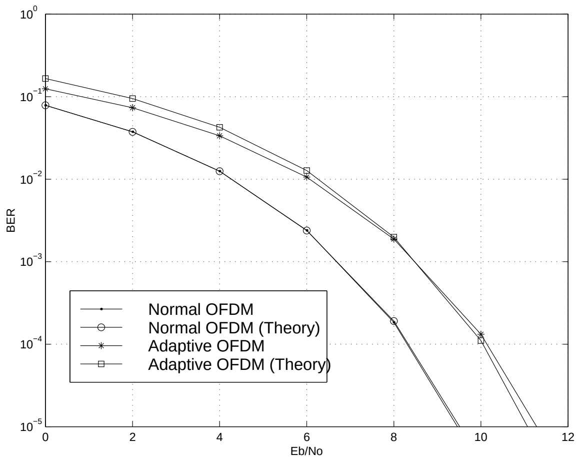

Fig. 5. BER of the Adaptive OFDM Scheme in AWGN channel for $M = 4$.

Fig. 5 shows the BER of an adaptive 4-ZPSK modulated OFDM signal in an AWGN channel. Maximum Likelihood (ML) coherent symbol detection is assumed. Further, we assume that the receiver has the exact knowledge of mapping used in the transmitter. The simulation results agree with theoretical evaluation using (14). The conventional modulation scheme (QPSK) performs better than that of 4-ZPSK scheme with a SNR gain of 2 dB at any BER. This is due to large minimum Euclidean distance in QPSK over 4-ZPSK.

IV. ICI Reduction

In M-ZPSK, we have a signal point of zero amplitude. Thus, some terms in the summation in (7) vanishes. Therefore, the adaptive M-ZPSK scheme is less sensitive to frequency offset errors than conventional schemes.

Fig. 6 shows the BER of an adaptive 4-ZPSK modulated OFDM signal with $\varepsilon = 0.1$. The adaptive scheme offers an SNR gain of 2 dB over conventional scheme at the BER of 10^{-3} in AWGN channel.

Interestingly, simplified adaptive M-ZPSK mapping could be used for ICI reduction if PAR reduction is not taken into account. In this case, the frequency of the bit pattern of $\log_2 M$ bits in an input data frame is counted. The most likely bit pattern is mapped to a signal constellation of zero amplitude. This increases the vanishing terms in the summation in (7) and thus

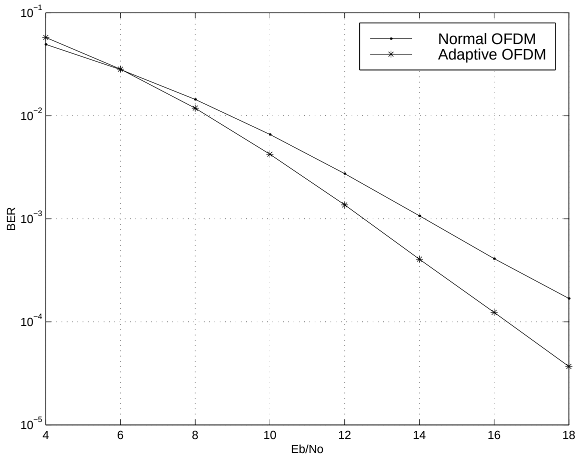

Fig. 6. BER of the Adaptive OFDM Scheme in AWGN channel for $M = 4$ with $\varepsilon = 0.1$.

reduces the ICI effects. Therefore, we need only one mapping and one IFFT calculation, as in the conventional system. A simple counter circuit can be used to select a mapping scheme based on the input data sequence. However, transmission of side information is necessary as in PAR reduction criteria. Note that this approach will not assure the PAR reduction but rather simple approach to reduce ICI effects. Therefore, we do not consider this simplified approach further as we are only interested in a single solution for PAR and ICI issues.

V. Impairments in the Presence of Nonlinearities

OFDM signal is subject to various hardware non-linearities in both the transmitter and receiver. Examples of these non-linearities are signal clipping in the analog to digital converter (A/D) converter, signal clipping in the IFFT and FFT processors with a limited word length, AM/AM and AM/PM distortion in the radio frequency (RF) amplifiers. Here, AM and PM stand for amplitude modulation and phase modulation respectively. Hardware non-linearities not only affect the performance of an OFDM system, but also may affect the system performance of an adjacent channel because of generated side lobes.

The effect of hardware non-linearities has been studied in [15]. Out of band power of OFDM signals increases, when amplified with nonlinear power amplifiers operating at lower back-offs. The high PAR of OFDM requires high back-offs at the amplifiers. The non-linearity present in the amplifier can be expressed by its AM/AM component $F[\rho]$ and AM/PM component $\Phi[\rho]$ [16], where ρ is the magnitude of the signal. In this paper, we will study the non-linearities caused by Solid-state power amplifiers (SSPA).

The input-output relationship of many SSPA can be modelled as

$$\begin{aligned} F[\rho] &= \frac{\rho}{[1+(\frac{\rho}{A})^{2P}]^{\frac{1}{2P}}} \\ \Phi[\rho] &= 0. \end{aligned} \tag{15}$$

where the parameter P controls the smoothness of the transition from the linear region to the limiting or saturation region. When $P \to \infty$, the SSPA model approximates the Soft Limiter (SL) characteristics. These models satisfy the non-expansive property and give a maximum output signal of A. The back off (BO) at the non-linear device can be defined in terms of A^2 and defined as

$$\mathrm{BO} = 10\log_{10}\left\{\frac{A^2}{E\{|x|^2\}}\right\} \tag{16}$$

where $E\{|x|^2\}$ is the average of the input power at the non-linear device. For PSD results, it is convenient to define the normalized frequency $B_n = fT$, where T is the OFDM symbol duration.

A. Out of Band Radiation

Out of band radiation can be caused by non-linearities. Power Spectral Density (PSD) of the OFDM signal and proposed adaptive OFDM signal is estimated via simulations. The PSD is estimated using Welch's averaged periodogram method with a Hanning window. An oversampling factor of 8 is used.

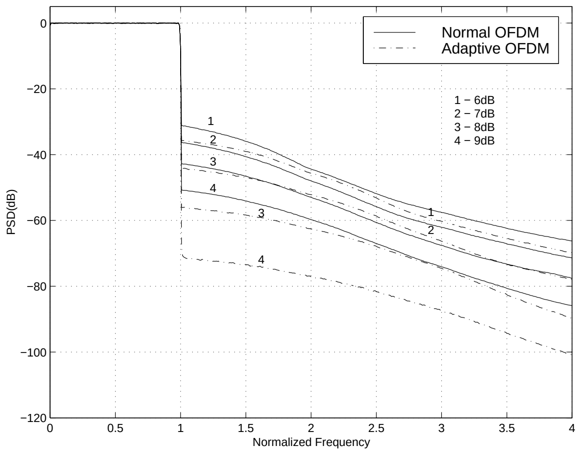

Fig. 7. Spectrum of an OFDM signal after passing through a SSPA.

Fig. 7 depict the PSD in the presence of SSPA nonlinear device. Curves 1 to 4 represent the different back-off values of the amplifier. For example, for a back-off of 8 dB, to provide -50 dB adjacent channel separation requires a channel spacing of about two times the symbol rate for normal OFDM, with adaptive OFDM the channel spacing is nearly the symbol rate. Thus, the adaptive OFDM provides a spectrally-efficient solution for the adjacent channel interference problem, which can arise in portable cellular applications. The performances in Fig. 7 reveals that depending on the permissible out of band radiation level, the amplifier back-off can be reduced by 2 to 3 dB when adaptive OFDM is used instead of conventional OFDM.

VI. Conclusion

In this paper, we have proposed an adaptive M-ZPSK to reduce both PAR and ICI of an OFDM signal simultaneously. The PAR improvement of the proposed scheme is comparable with other well known approaches [6, 7] with almost same or less complexity. Simulation and analytical results show that adaptive 4-ZPSK modulated OFDM system with 128 subcarriers improves the PAR by 2.5 dB at the expense of a 2 dB loss in SNR over conventional system. However, in a channel with normalized frequency offset of 0.1, it offers a 2 dB gain in SNR at BER of 10^{-3} over conventional scheme.

Moreover, for a back-off of 8 dB, to provide -50 dB adjacent channel separation requires a channel spacing of about two times the symbol rate for normal OFDM, with adaptive OFDM the channel spacing is nearly the symbol rate, which is a 50% saving in bandwidth. The price for these benefits is increased complexity over conventional OFDM systems.

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