Reducing Intercarrier Interference in OFDM Systems by Partial Transmit Sequence and Selected Mapping

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Abstract— **Orthogonal Frequency Division Multiplexing (OFDM) is sensitive to the carrier frequency offset (CFO). We introduce the Peak Interference-to-Carrier Ratio (PICR) to measure the resulting intercarrier interference (ICI). This paper shows that PICR can be reduced by Partial Transmit Sequence (PTS) and Selected Mapping (SLM) approaches. In PTS, each block of subcarriers is multiplied by a constant phase factor and these phase factors are optimized to minimize the PICR. In SLM, several independent OFDM symbols representing the same information are generated and the OFDM symbol with lowest PICR is selected for transmission. These schemes are analyzed theoretically and their performances are evaluated by simulation.**

Keywords— **Orthogonal Frequency Division Multiplexing, Carrier Frequency Offset (CFO), Intercarrier Interference (ICI)**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is an attractive technique for mitigating the effects of multipath delay spread of a radio channel. Therefore, it has been accepted for several wireless LAN standards, as well as a number of mobile multimedia applications [1]. Unfortunately, OFDM is sensitive to the CFO, which is caused by misalignment in carrier frequencies and/or Doppler shift. The CFO violates the orthogonality of subcarriers and results in ICI [1–4]. The BER performance degrades as a result.

The CFO issue is considered as a limiting factor in the commercial application of OFDM for high-rate mobile communications. This is particularly true where low-cost mobile hand sets that cannot employ very accurate frequency estimators because of the cost involved. In the open literature, several techniques have been proposed for reducing the ICI. These include frequency-domain equalization, time-domain windowing, correlative coding, self-ICI cancellation or polynomial coded cancellation and error correcting codes [2–4]. In this paper, we study the distribution for PICR as a measure for ICI effects. Interestingly, PICR is analogous to peak-to-average power ratio (PAR) issue for OFDM.

Controlling the Peak-to-Average Power Ratio (PAR) of an OFDM signal has gained a lot of attention recently [5–11]. Generating several statistically independent OFDM frames representing the same information sequence and selecting the one with the lowest PAR is a common approach [7–11]. This approach improves the statistics of the PAR of an OFDM signal with additional complexity. Motivated by the success of these approaches, we study the PTS [7–9] and SLM [7] approaches in this paper to reduce PICR.

In the PTS scheme, subcarriers are partitioned into blocks and each block is multiplied by a constant phase factor. These phase factors are optimized to minimize the PICR. Optimal phase factors are sent to the receiver as side information. In the SLM scheme, several independent OFDM symbols representing the same information are generated and the OFDM symbol with lowest PICR is selected for transmission. The independent OFDM symbols are generated by multiplying the information sequence by a set of fixed vectors. The receiver must know which multiplying vector has been used. A pointer to this sequence is transmitted as side information.

The organization of this paper is as follows: Section II presents the PICR problem in OFDM systems. PTS and SLM are explained in Section III. We explain the PTS approach to reduce PICR in Section IV. Section V presents the SLM approach ro reduce PICR. Simulation results are reported in Section VI and concluding remarks are presented in Section VII.

II. Peak Interference-to-Carrier Ratio (PICR)

A. *OFDM Signalling*

The complex baseband OFDM signal may be represented as

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} c_k e^{j2\pi k \Delta f t} \text{ for } 0 \le t \le T. \qquad (1)$$

where $j^2 = -1$ and N is the total number of subcarriers. The frequency separation Δf between any two adjacent subcarriers is equal to $1/T$ where T is the OFDM symbol duration. c_k is the data symbol for the k-th subcarrier. Each modulated symbol c_k is chosen from the set $F_q = \{\lambda_1, \lambda_2, .., \lambda_q\}$ of q distinct elements. The set F_q is called the *signal constellation* of the q-ary modulation scheme. The transmitted signal is modelled as Real$\{s(t)e^{j2\pi f_c t}\}$ where f_c is the carrier frequency.

We assume that $s(t)$ is transmitted on an additive white Gaussian noise channel, and so the received signal sample for the k-th subcarrier after Discrete Fourier Transform (DFT) demodulation can be written as [2]

$$y_k = c_k S_0 + \sum_{l=0, l \neq k}^{N-1} S_{l-k} c_l + n_k \text{ for } k = 0, ..., N-1 \qquad (2)$$

where n_k is a complex Gaussian noise sample (with its real and imaginary components being independent and identically distributed with variance σ^2). We shall refer to $\mathbf{c} = (c_0, c_1, ..., c_{N-1})$ as a data frame. The second term in (3) is the ICI term attributable to the CFO. The sequence S_k (the ICI coefficients) depends on the CFO and is given by [2]

$$S_k = \frac{\sin \pi(k+\varepsilon)}{N \sin \frac{\pi}{N}(k+\varepsilon)} \exp[j\pi(1-\frac{1}{N})(k+\varepsilon)] \quad (3)$$

where ε is the normalized frequency offset defined as a ratio between the frequency offset (which remains constant over each symbol period) and the subcarrier spacing. For a zero frequency offset, S_k reduces to the unit impulse sequence. The ICI on the k-th subcarrier can be expressed as (2)

$$I_k = \sum_{l=0, l\neq k}^{N-1} S_{l-k} c_l \text{ for } 0 \leq k \leq N-1. \quad (4)$$

Note that I_k is a function of both $\mathbf{c}$ and ε. In the sequel, we would be interested in reducing the peak magnitude of I_k.

B. PICR Problem

We define the Peak Interference-to-Carrier Ratio (PICR) as

$$\text{PICR}(\mathbf{c}) = \frac{\max_{0\leq k\leq N-1} |I_k|^2}{|S_0 c_k|^2}. \quad (5)$$

Note that the PICR is a function of both $\mathbf{c}$ and ε. PICR is the maximum interference-to-signal ratio for any subcarrier. In other words, it specifies the worst-case ICI on any subcarrier.

To reduce ICI effects, (5) should be minimized and is zero for ICI-free channels. Interestingly, our definition (5) is similar to PAR issue in OFDM [5–11]. However, the PICR problem differs from the PAR issue in several ways:

- ICI occurs at the receiver side, whereas high PAR values affect the transmitter.
- Exact computation of PAR requires oversampling, whereas max $|I_k|$ is obtained from N samples.
- As the transmitter does not know ε a priori, PICR can be computed only on the basis of a worst-case value, $\varepsilon_{wc} > 0$. Therefore, the performance of a PICR reduction scheme should hold for any $|\varepsilon| < \varepsilon_{wc}$.

Fig. 1 shows the complementary cumulative distribution function (CCDF) of the PICR as a function of N for $\varepsilon = 0.1$. The subcarriers are modulated with binary phase shift keying (BPSK). For $N = 128$, the PICR exceeds -4 dB for only 1 out 10^4 of all OFDM blocks. Therefore, the PICR can be considered as a random variable and is dependent on data. Naturally, one may apply PAR reduction techniques, which exploits the statistical properties of an OFDM signal, to reduce PICR. As PTS and SLM schemes can reduce PAR, we investigate them to reduce PICR.

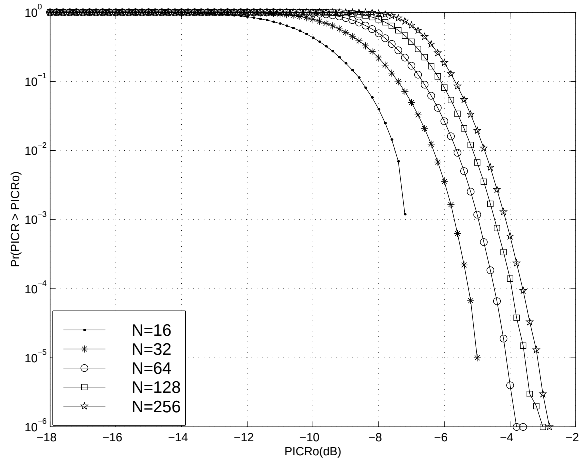

Fig. 1. CCDF of PICR of Normal OFDM System with $\varepsilon = 0.1$

III. PAR Reduction Schemes

A. Partial Transmit Sequences (PTS)

In PTS, the input data block is partitioned into disjoint subblocks or clusters which are combined to minimize the peaks. We define the data frame as vector, $\mathbf{c} = [c_0, c_1, ..., c_{N-1}]$ and partition it into M disjoint subblocks, represented by the vectors $\{\mathbf{c}_m, m = 1, 2, ..., M\}$, such that $\mathbf{c} = [\mathbf{c}_1, \mathbf{c}_2, .., \mathbf{c}_M]$. It is assumed that each subblock consists of a contiguous set of subcarriers and the subblocks are of equal size.

The objective of the PTS approach is to form a weighted combination of the M blocks,

$$\mathbf{c}_{new} = \sum_{m=1}^{M} b_m \mathbf{c}_m \quad (6)$$

where $\{b_m, m = 1, 2, ..., M\}$ are weighting factors and are assumed to be pure rotation. The Inverse Fast Fourier Transform (IFFT) of $\mathbf{c}_m$ are called Partial Transmit sequence. The phase factors are then chosen to minimize the peaks. The receiver must know the generation process of the transmitted OFDM signal. The phase factors must then be transmitted to the receiver as side information [7–9].

B. Selected Mapping (SLM)

U statistically independent alternative transmit sequences $\mathbf{a}^{(u)}$ represent the same information. The sequence with lowest PAR is selected for transmission. To generate $\mathbf{a}^{(u)}$, we define U distinct fixed vectors $\mathbf{P}^{(u)} = [P_0^{(u)}, ..., P_{N-1}^{(u)}]$ with $P_v^{(u)} = e^{j\varphi_v^{(u)}}$, $\varphi_v^{(u)} \in [0, 2\pi]$, $0 \leq v < N$, $1 \leq u \leq U$. Then, each modulated symbol $\mathbf{c} = [c_0, c_1, ..., c_{N-1}]$ is multiplied carrierwise with the U vectors $\mathbf{P}^{(u)}$, resulting in a set of U different modulated symbols $\mathbf{c}^{(u)}$ with components

$$c_v^{(u)} = c_v.P_v^{(u)} \text{ for } 0 \leq v < N \text{ and } 1 \leq u \leq U. \quad (7)$$

All U distinct modulated symbols are passed through IFFT process to get the transmit sequences $\mathbf{a}^{(u)} = \text{IFFT}\{\mathbf{c}^{(u)}\}$ and the sequence with lowest PAR is selected for transmission. In order to recover the data, the multiplied sequence has to be transmitted to the receiver as side information [7].

IV. ICI Reduction by Partial Transmit Sequence

As explained in the PTS approach, we partitioned the data frame $\mathbf{c} = [c_0, c_1, ..., c_{N-1}]$ into M disjoint subblocks and each subblock is zero padded to make its length N. Then, each subblock is multiplied by a weighting factor b_m $(m = 1, 2, ..., M)$. Thus,

$$I_{k,PTS} = \sum_{m=1}^{M} \sum_{l=0, l \neq k}^{N-1} b_m c_l^m S_{l-k} \quad (8)$$

where c_l^m is the data symbol in the newly formed m-th subblock.

Hence, (8) can be denoted as

$$I_{k,PTS} = \sum_{m=1}^{M} b_m I_k^m \quad (9)$$

where I_k^m is the interference on k-th subcarrier of block m. Thus, the total ICI is the weighted sum of ICI from each subblock. Therefore, ICI can be reduced by optimizing the phase sequence $\mathbf{b} = [b_1, b_2, ..., b_M]$. We optimize the phase factor to minimize the PICR. Therefore, this ICI reduction scheme is similar to PTS approach to reduce PAR.

The major drawback in PTS approach is the optimization of phase factor. To reduce the complexity of this optimization process, we only consider binary phase factors (i.e., $b_m = \pm 1$). Without loss of generality, we can set $b_1 = 1$ and observe that there are $(M-1)$ binary variables to be optimized. Finally, the optimal PICR can be found as

$$\text{PICR}_{optimal} = \min_{b_1,...,b_M} \left[\frac{\max_{0 \leq k \leq N-1} |I_{k,PTS}|^2}{|S_0 c_k|^2} \right]. \quad (10)$$

The computation of PICR at the transmitter requires ε. However, worst-case ε can be assumed for the computation. The simulation results show that computation of optimized phase factor is independent of ε if it is less than the worst-case scenario. This is an interesting and significant phenomenon.

V. ICI Reduction by Selected Mapping

The data frame $\mathbf{c} = [c_0, c_1, ..., c_{N-1}]$ is now multiplied symbol by symbol by a fixed vector $\mathbf{P}^{(u)} = [P_0^{(u)}, ..., P_{N-1}^{(u)}]$. For simple implementation, we select $P_v^{(u)} \in [\pm 1, \pm j]$ for $0 \leq v < N$, $1 \leq u \leq U$. Now, the resulting $I_{k,SLM}$ can be expressed as (Eqs. (4),(7))

$$I_{k,SLM} = \sum_{l=0, l \neq k}^{N-1} P_l^{(u)} c_l S_{l-k} \quad (11)$$

which is a function of the weighting sequence $\mathbf{P}^{(u)}$. Finally, the optimal PICR can be found as

$$\text{PICR}_{optimal} = \min_{\mathbf{P}^{(1)},...,\mathbf{P}^{(U)}} \left[\frac{\max_{0 \leq k \leq N-1} |I_{k,SLM}|^2}{|S_0 c_k|^2} \right]. \quad (12)$$

The worst-case ε can be assumed for the computation of PICR, as in the PTS approach.

VI. Simulation Results

Simulation results for PTS and SLM were obtained for an OFDM system with $N = 128$. The subcarriers are modulated with BPSK and an AWGN channel is assumed throughout this study. Further, a worst-case ε of 0.1 is assumed.

A. PTS Approach

Fig. 2 shows the CCDF of PICR for $M = 8$. In PTS approach, the PICR exceeds -6 dB for only 1 out 10^4 of all OFDM blocks whereas that of normal OFDM is for only 1 out 10. There is 2 dB reduction in PICR over normal OFDM with $M = 8$.

Optimized phase sequence requires 2^{M-1} computations of PICR. Note that an N-point IFFT and an N-point FFT are required at the transmitter to compute PICR. Thus, computation of optimized phase sequence is difficult. Instead, several selections of **b** can be generated randomly until PICR is reduced. Even 10 trials achieve a performance level that is nearly optimal.

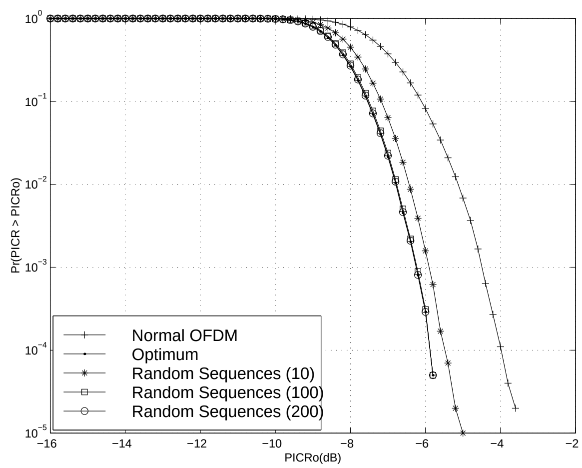

Fig. 2. CCDF of PICR of an OFDM System with $\varepsilon = 0.1$ and $M = 8$

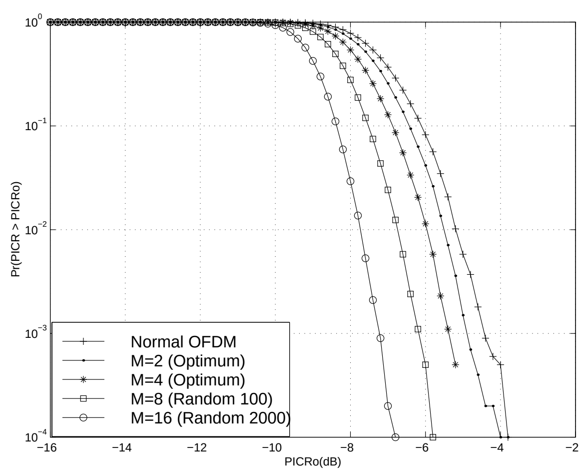

Fig. 3. Variation of CCDF of PICR with M for $\varepsilon = 0.1$

Fig. 3 shows how the performance varies with M. When M is large, PICR reduction is large. However, the computational complexity depends on M. Thus, it is a trade off between the performance and complexity.

In [8], a suboptimal iterative algorithm and Hadamard Walsh sequences were proposed as an alternative to the optimum phase sequence. In the suboptimal iterative algorithm, binary phase factors are flipped in an orderly manner to find the suboptimum sequence. As a first step, assume $b_m = 1$ for all m and compute PICR. Next, invert the first phase factor $(b_1 = -1)$ and recompute resulting PICR. If the new PICR is lower than the previous step, retain b_1 as part of the final sequence, otherwise, b_1 reverts to its previous value. The algorithm continues in this fashion until all M possibilities for ”flipping” the signs of the factors have been explored.

Fig. 4 shows the CCDF of PICR per OFDM for $\varepsilon = 0.1$ when 16 Walsh sequences of length 16 and the suboptimal iterative algorithm are used. 2000 random sequences performs better than Walsh and iterative algorithm whereas Walsh sequence performs better than iterative algorithm. With large number of random sequences, we can expect a near optimum performance with reasonable complexity.

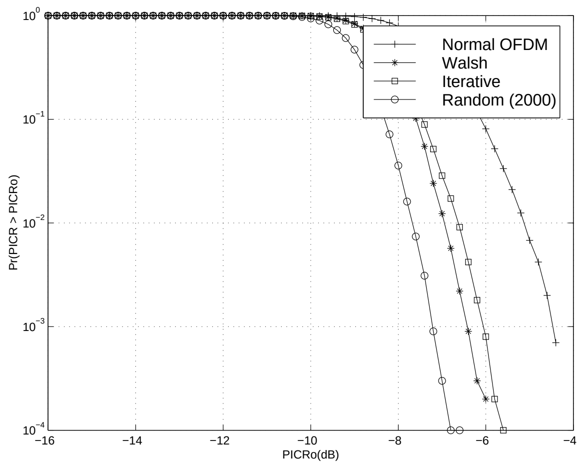

Fig. 4. CCDF of PICR of an OFDM System for different Phase Sequences with $M = 16$ and $\varepsilon = 0.1$

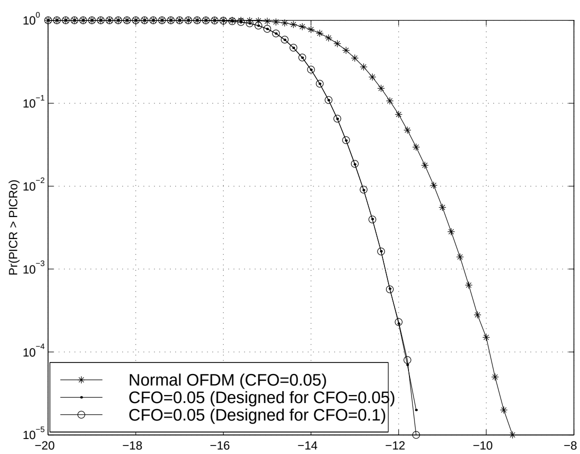

Fig. 5. CCDF of PICR of OFDM System for $M = 16$ and $\varepsilon = 0.1$

Fig. 5 shows the CCDF of PICR per OFDM block as a function of ε. PICR can be computed by assuming worst-case CFO and the optimization of phase sequence in PTS approach will not be affected by CFO less than the worst-case CFO.

B. SLM Approach

Fig. 6 shows the CCDF of PICR per OFDM block as a function of U for $\varepsilon = 0.1$. In SLM approach with $U = 8$, only 1 out 10^4 of all OFDM blocks exceeds the PICR of -8 dB whereas in normal OFDM, only 1 out 10^4 of all OFDM blocks exceeds the PICR of -3.5 dB. That is a 4.5 dB reduction in PICR. In fact, this is a significant reduction comparing with the PTS approach. Moreover, PICR reduction increases with increasing U. However, the computational complexity depends on U. Again, the performance can be traded off against complexity.

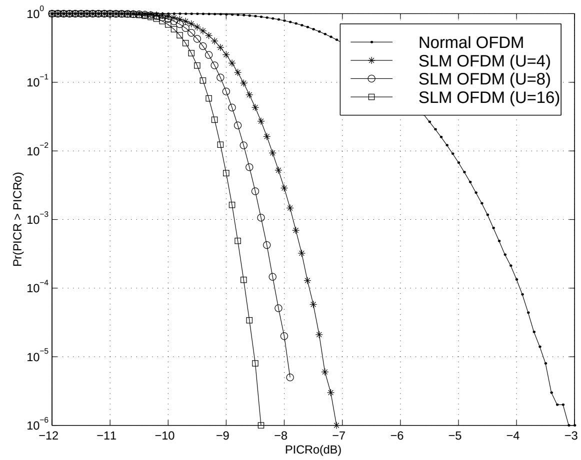

Fig. 6. CCDF of PICR of OFDM System with $\varepsilon = 0.1$

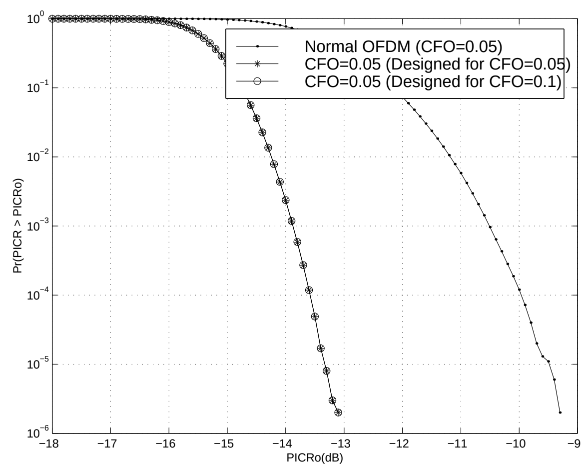

Fig. 7. CCDF of PICR of OFDM System with $U = 8$ and $\varepsilon = 0.1$

Fig. 7 shows that PICR can be computed by assuming worst-case CFO and the performance of the SLM approach will not be affected by CFO less than the worst-case CFO.

VII. Conclusion

In this paper, new solutions to the ICI problem in OFDM systems have been presented. The definition of PICR is analogous to that of PAR. Consequently, PAR reduction schemes can be applied to reduce PICR. We investigated the PTS and SLM methods to reduce PICR. They improves the PICR statistics of an OFDM signal at the expense of additional complexity, but with little loss in efficiency. For an OFDM system with $N = 128$ and $\varepsilon = 0.1$, PTS with $M = 16$ reduces PICR by 3 dB whereas SLM with $U = 16$ reduces PICR by 5 dB. Moreover, both schemes work independent of ε, provided $|\varepsilon| < \varepsilon_{wc}$.

REFERENCES

[1] R. van Nee, *OFDM wireless multimedia communications*. Boston, London: Artech House, 2000.

[2] J. Armstrong, "Analysis of new and existing methods of reducing intercarrier interference due to carrier frequency offset in OFDM," *IEEE Trans. Commun.*, vol. 47, pp. 365–369, Mar. 1999.

[3] J. Armstrong and C. Tellambura, "Multicarrier CDMA systems using PCC OFDM," in *IEEE VTC 2000*, vol. 2, pp. 1475–1479, IEEE, 2000.

[4] K. Sathananthan and C. Tellambura, "Forward error correction codes to reduce intercarrier interference in OFDM," in *IEEE ISCAS 2001*, (Sydney, Australia), IEEE, 2001.

[5] X. Li and L. J. Cimini, "Effects of clipping and filtering on the performance of OFDM," *IEEE Commun. Lett.*, vol. 2, pp. 131–133, May 1998.

[6] K. G. Paterson and V. Tarokh, "On the existence and construction of good codes with low peak-to-average power ratio," *IEEE Trans. Inform. Theory.*, vol. 46, pp. 1974–1987, Sept. 2000.

[7] S. H. Müller, R. W. Bäuml, R. F. H. Fisher and J. B. Huber, "OFDM with reduced peak-to-average power ratio by multiple signal representation," *Annals of Telecommunications*, vol. 52, pp. 58–67, Feb. 1997.

[8] L. J. Cimini and N. R. Sollenberger, "Peak-to-average power ratio reduction of an OFDM signal using partial transmit sequences," *IEEE Commun. Lett.*, vol. 4, pp. 511–515, Mar. 1999.

[9] C. Tellambura, "Improved phase factor computation for the PAR reduction of an OFDM signal using PTS," *IEEE Commun. Lett.*, vol. 5, no. 4, 2001.

[10] A. D. S. Jayalath and C. Tellambura, "Reducing the peak-to-average power ratio of an OFDM signal through bit or symbol interleaving," *IEE Elect. Lett.*, vol. 36, pp. 1161–1163, June 2000.

[11] A. Sumasu, T. Ue, M. Usesugi, O. Kato, and K. Homma, "A method to raduce the peak power with signal space expansion (ESPAR) for OFDM system," in *IEEE Vehicular Technology Conference*, (Piscataway, NJ, USA), pp. 405–409, IEEE, 2000.