IMPROVED TIME-DOMAIN EQUALIZER INITIALIZATION ALGORITHM FOR ADSL MODEMS

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Abstract-In this paper, we propose an improved adaptive initialization algorithm for the time domain equalizer in an FDM-based Asymmetric Digital Subscriber Line (ADSL). Using our developed approach, higher convergence rate than that of the commonly used Least-Mean Square algorithm is obtained, whilst attaining bit rates close to the optimum Maximum Shortening SNR and the upper bound Match Filter Bound (MFB). Furthermore, our proposed method outperforms the Minimum Mean-Squared Error design for a range of TEQ filter lengths. The improved performance outweighs the small increase in computational complexity.

1. Introduction

The evolution of multicarrier technology in the last decade has paved the way for the recent development of Digital Subscriber Line (xDSL) technology. In particular, ADSL has been the subject of much attention due to its widespread application in high-speed internet and video-on-demand systems. Discrete MultiTone (DMT) is used to partition the ADSL transmission bandwidth into N parallel sub-channels. The incoming bit streams are divided among the sub-channels in accordance with the receiver signal-to-noise ratio (SNR), and then each is QAM-modulated with complex sub-symbols. The complex sub-symbols are processed by an Inverse Fast Fourier Transform to form a real DMT time symbol. Prior to its transmission into the channel, a cyclic prefix is inserted at the beginning of the real DMT time symbol to combat Inter-Symbol Interference (ISI) between adjacent symbols and Inter-Carrier Interference (ICI) between successive samples of a symbol. If the length of the cyclic prefix is larger than the length of the equivalent discrete time channel or the channel impulse response (CIR), then ISI and ICI are effectively eliminated. However, in practical channels, the CIR can be of the order of hundreds of samples. In this case, a receiver equalization scheme comprising a T-tap time domain equalizer (TEQ) to shorten the length of the CIR and a frequency domain equalizer (FEQ) consisting of N 1-tap adaptive filters, to correct for signal phase rotation and signal amplitude attenuation, are used.

Many research investigations have been undertaken to define an effective TEQ initialization algorithm and over 20 different algorithms have been proposed. Both iterative and direct formula-based algorithms have been considered. Initialization algorithms can be classified into two main categories depending on the optimization method of the objective functions used, namely the minimum mean-squared error (MMSE) and the maximum shortening SNR (MSSNR) algorithms. Chow and Cioffi [1] first applied training-based MMSE equalization to multicarrier modulation, subject to a unit-tap constraint. Falconer and Magee [2] then introduced MMSE design in a decision-directed channel shortening equalization method for a maximum likelihood (ML) receiver, under a unit-energy constraint. Likewise, Van Bladel and Moeneclaey [3] have derived a TEQ from an MMSE viewpoint, requiring an eigenvalue solution. Al-Dhahir and Cioffi [4] subsequently generalized algorithms [1] and [3] and further showed that the MMSE constrained by a unit-energy bound outperforms the MMSE under a unit-tap constraint. In [2], Falconer and Magee also proposed an Least-Mean Square (LMS) algorithm bounded by a unit-energy constraint, whilst Chow, Cioffi and Bingham [5] presented an iterative algorithm that adapts the LMS [6] in the frequency domain and truncates and updates the TEQ and TIR in the time domain. LMS is simple to implement but its slow convergence behavior results in a long initialization period. On the other hand, the direct approach MMSE design involves matrix inversion and eigenvalue decomposition. This gives rise to high computational complexity, rendering real-time implementation impractical.

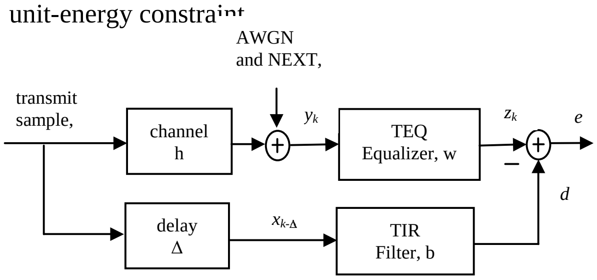
Melsa, Younce and Rohrs [7] proposed a MSSNR minimizing function, where the TEQ is designed to minimize the energy of the shortened impulse response at the outside of the target window, while maintaining constant energy inside. Although the MSSNR method outperforms MMSE-based techniques, it is equally computational intensive and not cost effective for real-time applications. This is because it too requires eigenvalue calculation, in addition to Cholesky decomposition. More recently, Acker *et al.* [8] devised a per-tone algorithm that optimizes the SNR on a per-tone basis. The key idea is to combine the T-tap TEQ and 1-tap FEQ into a single T-tap per tone frequency domain equalizer. The per-tone initialization algorithm [9] requires complex mathematics, thus the issue of complexity as encountered in both the MMSE and MSSNR designs remains largely unresolved by this approach.

In this paper, we propose a new iterative-based TEQ algorithm. Our approach applies a variant of the Least-Mean Fourth (LMF) algorithm [10] to the TEQ, with an LMS variant applied to the Target Impulse Response (TIR) filter. This combined TEQ structure results in a useful speed-up in the convergence rate, whilst providing a significant improvement in the SNR performance. This is achieved at a relatively low computational overhead. The rest of this paper is organized as follows. Section 2 provides a brief overview of TEQ channel shortening. Section 3 then describes our proposed algorithm. The results obtained are discussed in section 4, while conclusions are presented in section 5.

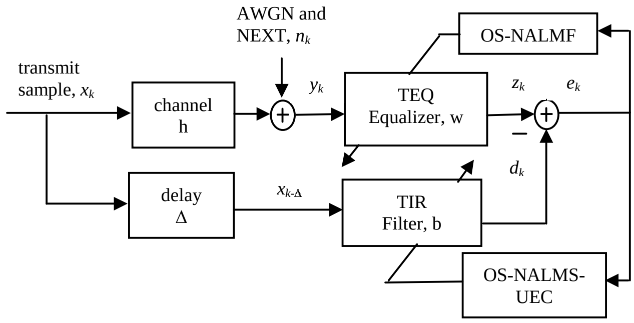
2. TEQ Channel Shortening Equalization- A Brief Overview

The TEQ channel shortening equalization approach used [1] is illustrated schematically in fig. 1a. Here the goal is to minimize the mean-squared error between the output of the TEQ equalizer and the TIR filter, when both are trained with the MMSE algorithm. As depicted in figure 1a, the minimization of the mean square error, e_k, involves the search for a range of synchronization delays, Δ. To minimize the initialization period, a fast converging algorithm to limit the search period for each Δ is essential.

An important aspect of our proposed algorithm has been to recognize that the initialization algorithm for the TEQ equalizer should be capable of rejecting ISI and ICI in the received ADSL signal. The conventional LMS is well known for its simplicity and robustness. However, it suffers performance degradations when the received signal in the TEQ is distorted by ISI, ICI and, to a certain degree, by impulse noise in a hostile environment. In order to alleviate this problem, we use an Order Statistics (OS) LMS and LMF [10] approach for TIR and TEQ adaptation respectively, with the TIR subject to a unit-energy constraint.



(a)



(b)

Figure 1. Model for TEQ channel shortening equalization (a) conventional MMSE structure (b) proposed hybrid structure.

3. The Proposed Algorithm

Walach and Widrow [10] showed that the LMF algorithm is particularly suited for applications in non-Gaussian noise conditions with lower weight noise than the LMS algorithm. Chan and Cowan [11] have also shown that the normalized LMF exhibits a faster convergence rate. Our investigations have shown that when this is applied to TEQ adaptation, the results are not promising. In order to improve this, we therefore developed an OS modification to the LMF using a transformation technique originally applied to LMS by Clarkson and Harweel [12]. This was later improved by Chambers [13]. We have therefore developed a new OS Normalized Averaged LMF (OS-NALMF) algorithm for TEQ adaptation. The TIR filter, on the other hand, is driven by a delayed version of the transmitted symbols, $x_{k-\Delta}$, that are supposedly ISI and ICI-free. In this case, we have therefore used the less complex OS Normalized Averaged LMS (OS-NALMS) algorithm. This hybrid approach developed is illustrated in fig. 1b.

The LMF algorithm. The LMF uses the mean-fourth cost function to search for the global minimum. The power-of-four cost function of the LMF is given as,

$$J_{\mathrm{LMF},k} = \mathrm{E}[e_k^4] = \mathrm{E}[\{d_k - z_k\}^4] \qquad (1)$$

where the error, e_k ,at time instant, k, is the difference between the output of the TIR filter, d_k ,and the TEQ output, z_k (fig. 1). The method of steepest descent is used to search for the minimum point on the mean-fourth error performance surface leading to a gradient vector of

$$\frac{\partial}{\partial \mathbf{w}_k} J_{\mathrm{LMF},k} = 4e_k^3 \mathrm{E}\left[\frac{\partial}{\partial \mathbf{w}_k}\{d_k - \mathbf{y}_k \mathbf{w}_k\}\right] = -4e_k^3 \mathbf{y}_k \quad (2)$$

Using equation (2), the filter coefficient update of the least-mean fourth algorithm is given as,

$$\mathbf{w}_{k+1} = \mathbf{w}_k + 4\,\mu_w\, e_k\, \mathbf{y}_k \qquad (3)$$

The Order Statistics (OS) Transformation. The OS modification is derived by substituting a non-linear operation for the linear smoothing of the instantaneous gradient estimate of the LMF. The non-linear function is simply an algebraic ordering of the gradient estimate. The update equation of the OS-LMF algorithm for the N_w–tap TEQ is given as

$$\mathbf{w}_{k+1} = \mathbf{w}_k + 4\,\mu_w\, \mathbf{M}_{k,w}\, \boldsymbol{a}_w \qquad (4)$$

where $\mathbf{M}_{k,w} = T\{e_k^3 \boldsymbol{y}_k\}$ is a N_w by L matrix and T denotes the algebraic ordering transformation applied to the sequence of gradient estimates $\{e_k^3 \boldsymbol{y}_k,\ e_{k-1}^3 \boldsymbol{y}_{k-1}, \ldots,\ e_{k-L+1}^3 \boldsymbol{y}_{k-L+1}\}$, $\boldsymbol{a}_w = (a_1, a_2, \ldots, a_L)$ is a vector of weighting factors, $\mathbf{w}$ is the vector of TEQ coefficients , μ_w is the TEQ adaptation step-size, L is the OS window length and k is the time index. The weighting vector, $\boldsymbol{a}$, is referred to as the smoothing OS filter. Depending on $\boldsymbol{a}$, a number of special classes of OS-LMF algorithms, such as the averaged and the median transformations can be derived independently. The algebraic ordering transformation simply involves sorting out the elements of the gradient vector in an ascending order. A "median" selection of the gradient vector gives rise to the non-linear behavior of the OS algorithm. Here, we focus on the averaged LMF (ALMF) [12] by setting the weighting factor as the average of the gradient vector estimate, i.e.

$$a_i = \frac{1}{L} \qquad (5)$$

where i = 1, 2,…, L. The algebraic ordering transformation is excluded from the ALMF algorithm as the gradient vector does not need to be ordered in an averaging process. The averaged transformation is an OS algorithm that coincides with linear operation. In order to improve the performance of the ALMF algorithm in non-stationary environments, the step-size μ_w is normalized by the input signal power. However, instead of computing the estimates of the input power using the squared Euclidean norm of the N_w taps for normalization, we use equation (6), which requires less computational effort.

$$\hat{P}_{y,k+1} = \hat{P}_{y,k} + \mathbf{y}_k^2 - \mathbf{y}_{k-N_w+1}^2 \qquad (6)$$

Here $\hat{P}_{y,k}$ is the estimate of the input signal power at time instant k. Equations (4-6) describe the OS-NALMF algorithm for TEQ equalizer. The TIR filter update is performed with the OS-NALMS, approach given as

$$\mathbf{b}_{k+1} = \mathbf{b}_k - \mu_b\, \mathbf{M}_{k,b}\, \boldsymbol{a}_b \qquad (7)$$

Here $\mathbf{M}_{k,b} = T\{e_k \boldsymbol{x}_{k-\Delta}\}$ is the algebraic ordering transformation of the N_b by L gradient estimate $\{e_k \boldsymbol{x}_{k-\Delta},\ e_{k-1} \boldsymbol{x}_{k-\Delta-1}, \ldots,\ e_{k-L+1} \boldsymbol{x}_{k-\Delta-L+1}\}$, $\boldsymbol{a}_b = (a_1, a_2, \ldots, a_L)$ is a vector of weighting factors, $\mathbf{b}$ is the vector of TIR coefficients , μ_b is the TIR adaptation step-size, N_b is the number of TIR taps, L is the window length and k is the time index. Equivalently, the input power normalization is given as $\hat{P}_{x,k+1} = \hat{P}_{x,k} + \mathbf{x}_k^2 - \mathbf{x}_{k-N_b+1}^2$. The coefficient $\mathbf{b}$ vector is subject to a unit-energy constraint in order to avoid convergence to a trivial solution. Hence, $\mathbf{b}$ is normalized with respect to its Euclidean norm, given as

$$\mathbf{b}_{k+1} = \frac{\mathbf{b}_{k+1}}{\left\|\mathbf{b}_{k+1}\right\|} \qquad (8)$$

4. Simulation Results

In this section, we discuss simulation results for our proposed hybrid algorithm. This we denote as algorithm A. For the purpose of performance comparisons, we have included an MMSE design [4], an MSSNR design [7], a hybrid OS-NALMS/LMS (denoted as algorithm B) and the conventional LMS/LMS algorithm (denoted as algorithm C). For these investigations, we considered the frequency division multiplex (FDM) based downstream ADSL channel, CSA loop #6. The channel is modeled with a 512-tap zero phase FIR (without transmit and receive filters). Simulation specifications are given in table 1.

Simulations were run for a range of delays, with the delay corresponding to the lowest mean-squared error being chosen as the optimal delay. In each case the performance of the received SNR, achievable bit rates and the convergence behavior of the iterative-based algorithms was measured. The SNR achieved with respect to the MFB for the five different algorithms is plotted in Figures 2(a)-(e). The results obtained show that the MMSE method (fig. 2a) suffers from deep notches at frequencies across the transmission bandwidth. Similar effects can be viewed with the algorithm C (fig. 2e). Conversely, the SNR performance of algorithm A (fig. 2c) matches with the algorithm B (fig. 2d) and exhibits very close resemblance to that of the MSSNR method (fig. 2b), apart from the very low frequency region. ADSL downstream bandwidth ranges from tone 32 (138 kHz) and above. Thus the variations in SNR in the low frequency region are of little relevance. Fig. 3 shows the convergence behavior of algorithms A, B and C. From these curves, algorithm A shows the fastest speed, followed by algorithm B whilst C shows relatively slow convergence. Figure 4 depicts the achievable bit rate as a function of the TEQ filter length. The results show that algorithm A produces bit rates to within 99% of the MSSNR, 98% of the MFB and marginally outperforms the MMSE design for a range of TEQ taps. Extensive simulations were conducted to search for an optimal combination of step sizes, μ_w and μ_b. The results show that algorithm A does not suffer from instability under an optimal step size condition. We have also derived the computational complexity of algorithm A for a fixed Δ and compare it with that of the algorithm C. Both TIR algorithms are bounded by the unit-energy constraint. Table 2 shows that algorithm A an increase of N_b+N_w+4 multiplications, two divisions and $(L-1)(N_b+N_w)+4$ additions/subtractions per iteration when compared with algorithm C.

5. Conclusions

In this paper, we have presented an improved adaptive algorithm for TEQ initialization. We studied the effect of combining the OS-NALMF and OS-NALMS algorithms for TEQ and TIR initialization respectively in an FDM-based ADSL channel shortening equalization. The results obtained show that this gives a very fast convergence rate and a near MFB SNR. Furthermore our proposed hybrid structure achieves bit rates to within 99% of MSSNR and 98% of MFB performance. It also outperforms the MMSE design for a wide range of TEQ filter sizes. This finding provides a viable solution to overcome the limitations associated with slowly converging algorithms, such as the LMS, and computational intensive algorithms, such as the MMSE and MSSNR, whilst at the same time delivering a performance close to optimum without significant sacrifices in computational costs. The fast convergence behavior of our algorithm enhances the prospect of fast re-training and widens the scope of search for the optimum delay without trading-off modem start-up time.

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TABLE I
SIMULATION SPECIFICATIONS

Parameters	Specifications
FFT size (N)	512
TEQ length (Nw)	20
TIR length (Nb)	33
Input signal power (dBm)	14
Additive white gaussian noise (dBm/Hz)	-113
Sampling rate (MHz)	2.208
SNR gap (dB)	9.8
Noise margin (dB)	6
Coding gain (dB)	4.2
Near-End crosstalk (NEXT)	10

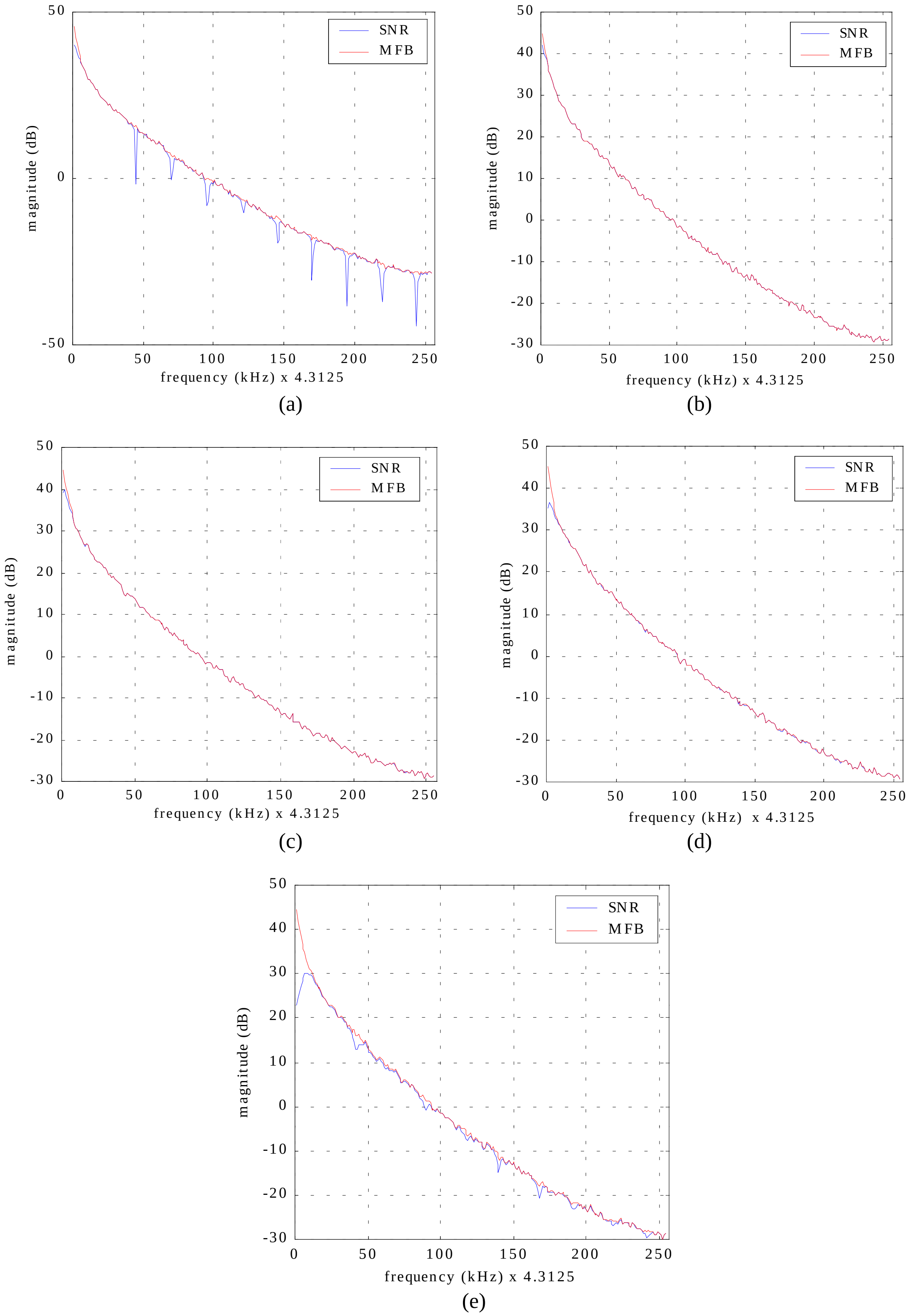

Figure 2. SNR performance vs. frequency bins of TEQ initialization algorithms
(a) MMSE [4], (b) MSSNR [7], (c) Algorithm A, (d) Algorithm B, (e) Algorithm C.

TABLE II
COMPUTATIONAL COMPLEXITY OF LMS, OS-NALMS, OS-NALMF and the UEC.
($\times$)-multiply, ($\div$)-divide, ($\sqrt{\ }$)-square-root, (+/-)-add/subtract

Algorithm	($\times$)	($\div$)	($\sqrt{\ }$)	(+/-)
LMS	N_b^*+1	0	0	N_b^*
OS-NALMS	$2N_b^*+2$	1	0	N_b^*L+2
OS-NALMF	$2N_w+4$	1	0	N_wL+2
UEC	N_b	1	1	N_b
Algorithm A	$3N_b+2N_w+6$	3	1	$N_wL+(L+1)N_b+4$
Algorithm B	$2N_b+2N_w+3$	2	1	$2N_b+N_wL+2$
Algorithm C	$2N_b+N_w+2$	1	1	$2N_b+N_w$

$^*N_w$ if LMS is used for TEQ.

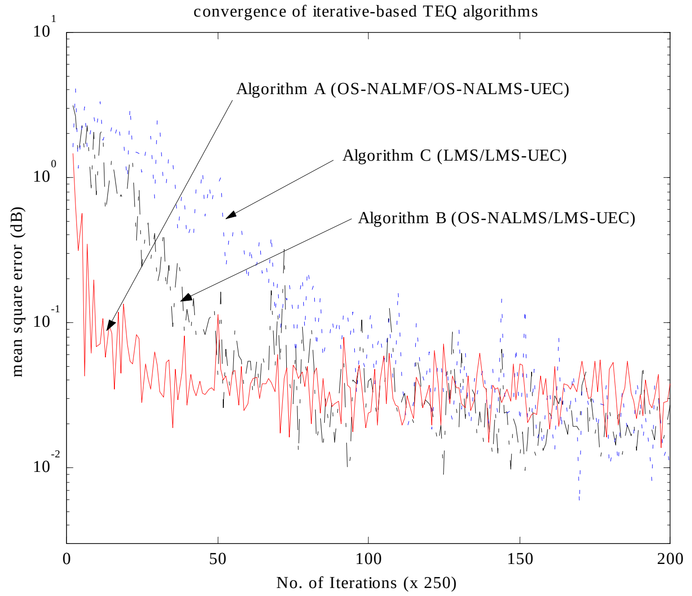

Figure 3. Convergence of iterative-based TEQ initialization algorithms.

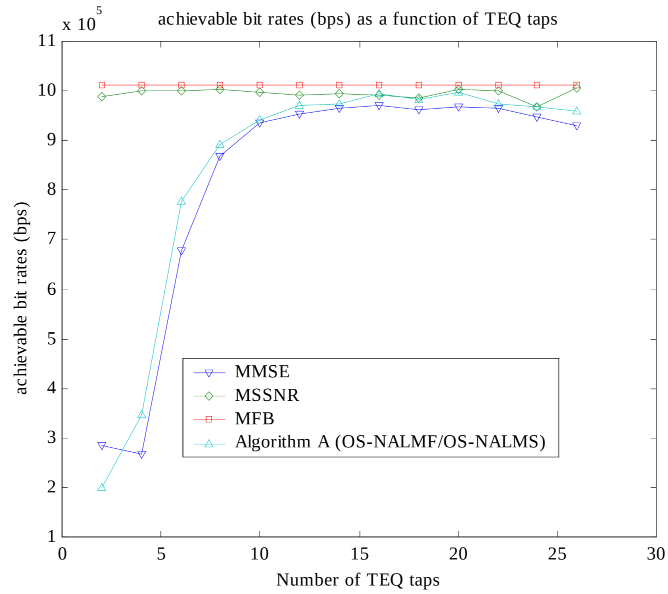

Figure 4. Achievable bit rates (bps) vs. TEQ tap (N_w) of TEQ initialization schemes.