DS CDMA Scheme for WLANs with Errors and Erasures Decoding

B.J.Wysocki, T.A.Wysocki
University of Wollongong
Northfields Av., NSW 2522
Australia
wysocki@uow.edu.au

Abstract - In the paper, we present simulation results for the 32 channel DS CDMA WLAN utilising modified Walsh-Hadamard spreading sequences. The method to obtain those spreading sequences is shown. The resultant system BER as well as the distribution of errors within frames is given. The obtained results indicate that with the application of a hybrid ARQ scheme with errors and erasures decoding, the number of frames, which would require retransmission is significantly lower than when the hard decoding is employed. Another benefit of using errors and erasures decoding is increase in the system reliability with a 2/3 drop in the number of undetected errors.

1. Introduction

During the 1990s, direct sequence spread spectrum code division multiple access (DS CDMA) technology [1] has matured as a technique to provide multiple access to the radio channel for mobile communications. For example, it is used in mobile telephony compliant with IS-95 standard [2]. The major benefits of DS CDMA is that it can be effective in combating problems related to multi-path propagation, while providing good interference from other, narrowband devices operating in the same frequency band. This is, however, dramatically reduced if only a small processing gain [1] can be achieved, as in the case of WLANs. Under such conditions, in-band jammers, like other channels of the same WLAN acquired by means of CDMA cause severe multiple access interference (MAI), which may block the communication. In theory, cancellation of that type of interference is possible if each of the users utilize orthogonal signals to transmit the data [1]. If delays between transmitters and receiver are anyhow different, as is generally the case of terminal to base station (BS) transmission, the signals received by the BS cannot be regarded as orthogonal. Within the 50m-coverage area those differences may be in the order of a few spreading code symbols (chips) depending on the data rate. This effect is particularly critical for very short spreading sequences, like 32-bit Walsh functions.

2. Sequence design method

Sets of spreading sequences used for DS CDMA applications can be represented by MxN matrices $\mathbf{S}_{MN}$, where M is the number of sequences in the set and N is the sequence length. The sequences are referred to as orthogonal sequences if, and only if the matrix $\mathbf{S}_{MN}$ is orthogonal, i.e.

$$\mathbf{S}_{MN}\mathbf{S}_{MN}^{T} = k\mathbf{I}_{M} \quad (1)$$

where k is a constant, $\mathbf{S}_{MN}^{T}$ is the transposed matrix $\mathbf{S}_{MN}$, and $\mathbf{I}_{M}$ is an MxM unity matrix.

There are a few families of orthogonal spreading sequences proposed in literature, e.g. [3], [4], [5], [6], and [7]. Out of them, the most commonly applied are Walsh-Hadamard sequences. Some of the proposed sequence families are designed in a parametric way, which allows for some manipulation of parameters to change the desired correlation characteristics. However, those changes are usually of a limited magnitude, and very often while improving the crosscorrelation functions, a significant worsening of the autocorrelation functions is experienced, e.g. [8].

In [9], we proposed to modify correlation properties of the set of orthogonal spreading sequences by multiplying the matrix $\mathbf{S}_{MN}$ by another orthogonal $N \times N$ matrix $\mathbf{D}_{N}$. The new set of spreading sequences is then represented by a matrix $\mathbf{W}_{MN}$

$$\mathbf{W}_{MN} = \mathbf{S}_{MN}\mathbf{D}_{N} \,. \quad (2)$$

The matrix $\mathbf{W}_{MN}$ is also orthogonal, since:

$$\mathbf{W}_{MN}\mathbf{W}_{MN}^{T} = \mathbf{S}_{MN}\mathbf{D}_{N}(\mathbf{S}_{MN}\mathbf{D}_{N})^{T} = \mathbf{S}_{MN}\mathbf{D}_{N}\mathbf{D}_{N}^{T}\mathbf{S}_{MN}^{T} \quad (3)$$

and because of the orthogonality of matrix $\mathbf{D}_{N}$, we have

$$\mathbf{D}_{N}\mathbf{D}_{N}^{T} = c\mathbf{I}_{N} \quad (4)$$

where c is a real constant. Substituting (4) into (3) yields

$$\mathbf{W}_{MN}\mathbf{W}_{MN}^{T} = c\mathbf{S}_{MN}\mathbf{I}_{N}\mathbf{S}_{MN}^{T} = c\mathbf{S}_{MN}\mathbf{S}_{MN}^{T} = kc\mathbf{I}_{M} \,. \quad (5)$$

In addition, if $c = 1$, then the sequences represented by the matrix $\mathbf{W}_{MN}$ are not only orthogonal, but possess the same normalization as the original sequences represented by the matrix $\mathbf{S}_{MN}$. However, other correlation properties of the sequences defined by $\mathbf{W}_{MN}$ can be significantly different to those of the original sequences.

To this point, it is not clear how to chose the matrix $\mathbf{D}_{N}$ to achieve the desired properties of the sequences defined by the $\mathbf{W}_{MN}$. In addition, there are only a few known methods to construct the orthogonal matrices, such as those used for the Hadamard matrices [3]. However, another simple class of orthogonal matrices are diagonal matrices with their elements $d_{m,n}$ fulfilling the condition:

$$\left|d_{m,n}\right| = \begin{cases} 0 & \text{for} \quad m \neq n \\ c & \text{for} \quad m = n \end{cases}; \quad m, n = 1, \ldots, N \quad (6)$$

To preserve the normalization of the sequences, the elements of $\mathbf{D}_N$, being in general complex numbers, must be of the form:

$$d_{m,n} = \begin{cases} 0 & \text{for} \quad m \neq n \\ \exp(j\phi_m) & \text{for} \quad m = n \end{cases}; \quad m, n = 1, \ldots, N \tag{7}$$

where the phase coefficients ϕ_m; m = 1, 2, …, N, are real numbers taking their values from the interval [0, 2π), and $j^2 = -1$. The values of ϕ_m; m = 1, 2, …, N, can be optimized to achieve the desired correlation and/or spectral properties, e.g. minimum out-off-phase autocorrelation or minimal value of peaks in aperiodic crosscorrelation functions.

From the implementation point of view, the most important class of spreading sequences are bipolar or biphase sequences, where the ϕ_m; m = 1, 2, …, N, can take only two values 0 and π. This results in the elements on the diagonal of $\mathbf{D}_N$ being equal to either '+1' or '-1'. Even for this bipolar case, we can achieve significantly different properties of the sequences defined by the $\mathbf{W}_N$ than those of the original Walsh-Hadamard sequences of the same length.

As an example, let us compare some properties of Walsh-Hadamard sequences, with the properties of the sequence set defined by the bipolar matrix $\mathbf{W}_{32}$, with the diagonal of the $\mathbf{D}_{32}$ represented for the simplicity by a sequence of '+' and '-' corresponding to '+1' and '-1', respectively:

{++++---+++--+-+-+--++-+---+----+} (11)

For the unmodified set of 32 Walsh-Hadamard sequences of length N = 32, the maximum in the aperiodic crosscorrelation function [3] ($ACC_{\max}$) reaches 0.9688, and the mean square out-of-phase aperiodic autocorrelation [3] (*MSAAC*) is equal to 6.5938. That high value of *MSAAC* indicates the possibility of significant difficulties in the sequence acquisition process, and the high value of $ACC_{\max}$ means that for some time shifts the interference between the different DS CDMA channels can be unacceptably high. On the other hand, for the set of sequences defined by the matrix $\mathbf{W}_{32}$ considered here, we have $ACC_{\max}$ = 0.4375, and *MSAAC* = 0.8438. This means lower peaks in the instantaneous bit-error-rate due to the MAI and a significant improvement in the sequence acquisition process.

In Fig. 1, we present the plot of ACC_{max} as a function of the relative shift between the sequences for the modified sequence set. It is clearly visible that the method preserves their orthogonality for the perfect synchronization, while significantly reduces peaks in their mutual cross-correlation functions. In addition, the sequences designed in that way are characterized by much better autocorrelation functions than the original Walsh-Hadamard sequences, as is shown in Fig. 2. As result, the sequence acquisition can be performed easier and more reliably.

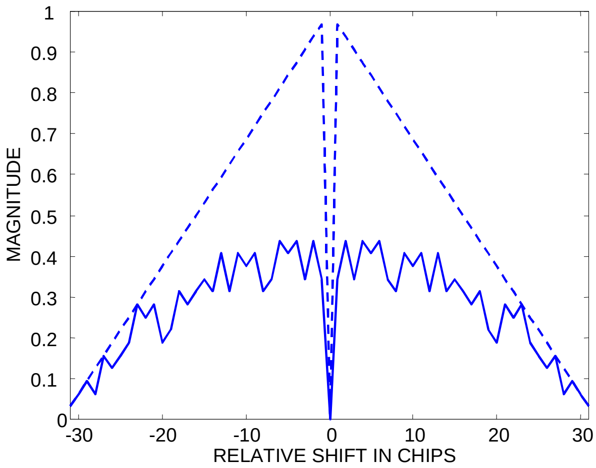

Figure 1: Plots of the maximum value of the aperiodic cross-correlation ACC_{max} for all possible pairs of the sequences versus the relative shift between them; 32-chip modified Walsh-Hadamard sequences - solid line, 32-chip pure Walsh-Hadamard sequences – dashed line.

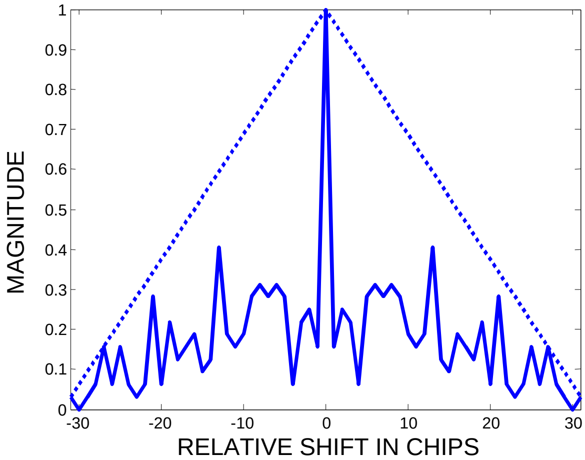

Figure 2: Plots of the maximum value of the aperiodic auto-correlation AAC_{max} for all possible pairs of the sequences versus the relative shift between them; 32-chip modified Walsh-Hadamard sequences - solid line, 32-chip pure Walsh-Hadamard sequences – dashed line.

3. Simulation of DS CDMA wireless LAN

For the protection of transmitted data, we have applied a shortened to 424 information bits (1023,923) BCH code. It can correct up to 10 random errors and fill in 21 random erasures. The simulation reported here consisted of the transmission of 500 code words carrying the same random information, spread using the 32-chip orthogonal sequences designed according to the proposed method. The modulation used was BPSK, and for each transmitted frame there were 15 randomly selected interferers. Each of the interfering signals contained a random data, encoded using the same shortened BCH code. The interferers were not synchronized with the desired signal, i.e. there was a random $0 \le \tau \le 31$-chip shift between the starting points of the sequences. In addition, the phases of RF modulators were assumed random. The selection of interferers was repeated for each of the transmitted frames. The whole simulation was repeated for each of the 32 possible channels. As an example, in Fig. 2, we present a histogram of the number of errors in received frames for a hard decision correlator receiver, before decoding has been applied.

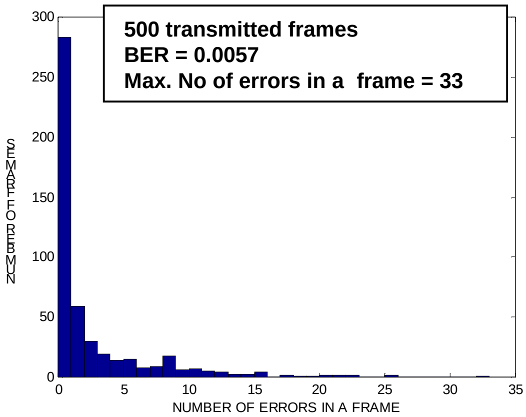

Figure 2: Histogram of a number of errors in a transmitted frame for channel 32.

The received frames were then decoded using first the hard decision decoding. The average BER at the output of a hard decision decoder was $4.1*10^{-3}$ compared to $7.6*10^{-3}$ before decoding.

The decoding was later repeated utilizing an errors and erasure decoding algorithm [10]. The received code words at the output of an erasure decision demodulator may contain erasures, which positions are known but the symbol values are unknown. In addition to the erasures, there may be also errors in the received code words, where neither positions nor symbol values are known. The errors-and-erasures decoder has to fill in proper symbol values for the erasures and correct the errors. The applied errors-and-erasures decoder [10] works according to the following algorithm:

- Compute an erasure polynomial base on known erasure locations.
- Initially replace erasures with zeros and compute the syndromes.
- Compute a modified syndrome polynomial.

- Apply Berlekamp-Massey algorithm to find coefficients of error locator polynomial using the modified syndrome.
- Apply Chien search algorithm to find the roots of error locator polynomial.
- Form the error and erasure locator polynomial.
- Determine symbols for erasures and correct errors.

One of the most important issues, while performing the errors-and-erasures decoding, is setting the proper value for the erasure threshold. If the threshold value is too high, there are too many erasures, which in fact could be correctly decoded by a hard decision decoder. If it is too low, then the performance of the decoder is similar to that of the hard decision. From our simulations, it could be seen that the performance of the decoder varies significantly with the value of the erasure threshold. However, for most of the channels, there was a significant improvement in the BER performance for a fairly wide range of the threshold between 0.02 and 0.08, as far as the normalized value of the correlator receiver output is concerned. An example plot of the BER for a single channel as a function of the threshold value is given in Fig. 3.

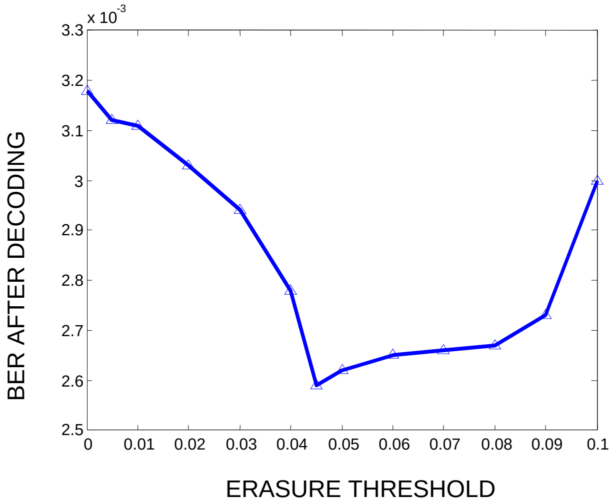

Figure 3: BER versus the erasure threshold value for the channel 21.

The operation of errors and erasures decoding has been performed for all 32 channels, and in all cases we have achieved a significant improvement in BER compared with the hard decoding algorithm. For the comparison, the results obtained with the optimum thresholds are plotted in Fig. 4 for all 32 channels.

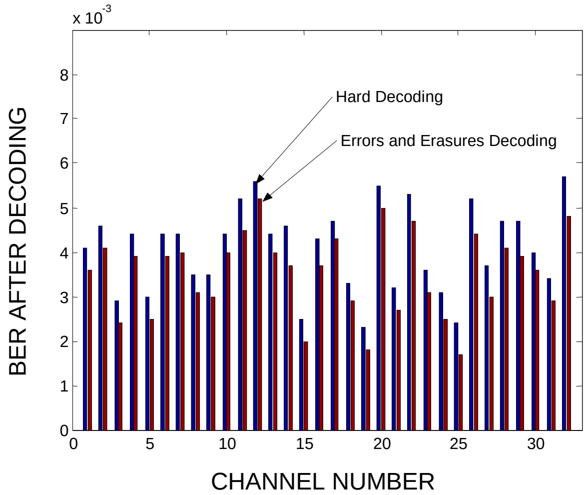

Figure 4: Improvement in BER for errors and erasures decoding compared with hard decision decoding for all 32 channels; for all channels the optimal threshold have been applied.

By applying the errors and erasures decoding, we achieved an average BER after decoding equal to $3.5*10^{-3}$. As a result, the number of code words requiring retransmission was in average reduced by 13%. More importantly, however, the number of undetected errors dropped 3 folds for errors and erasures decoding, i.e. from 15 cases to only 5.

4. Conclusions

In the paper, we presented simulation results for the asynchronous DS CDMA WLAN utilizing the modified Walsh-Hadamard spreading sequences of length 32. The results show that for the developed sequence set, a significant number of active users can be accommodated if the hybrid ARQ employing shortened BCH code, and errors and erasures decoding is employed.

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