CHAPTER

Polyphase Chirp Sequences for Wireless Applications

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1. INTRODUCTION

There are several families of binary and complex spreading sequences proposed in literature [1], [2], [3], [4], [6], [7], with some of them, e.g. sequences proposed in [6] allowing for a good compromise between mean square aperiodic crosscorrelation and mean square aperiodic autocorrelation for the whole set. There are, however, no clear ways how to chose appropriate values of parameters to achieve the desired spectral characteristics, and to avoid high peaks in crosscorrelation functions (CCFs). In this paper, we propose a method to design a useful set of sequences for DS CDMA wireless data networks. Based on the fact that use of complex spreading codes introduces a phase modulation into the band-pass signal, we look into the properties of sequences obtained on the basis of a linear combination of baseband chirps [9], which are among the analogue signals having very good autocorrelation properties. The similar approach has been used by Popovic [7] in design of his P3 and P4 sequences, utilising a single-chirp like sequences. Here, however, we will look into design of sequences for any given length *N*. Ability to do so, is very important from the viewpoint of applying such sequences in wireless data networks for variable data rates

2. CHIRP SEQUENCES

2.1 Design method

Chirp signals are widely used in radar applications for pulse compression [10], and were also proposed for use in digital communications by several authors, e.g. [11]. They refer to creation of such a waveform where an instantaneous frequency of the signal changes linearly between the lower and upper frequency limits. This is graphically illustrated in Figure 1, which presents the two basic types of chirp pulses and their instantaneous frequency profiles.

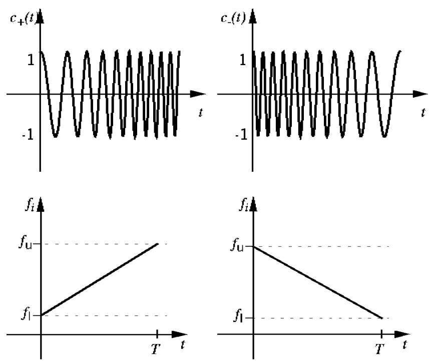

Figure 1: Positive and negative chirp pulses and their instantaneous frequency profiles.

For the positive chirp $c_+(t)$, the instantaneous frequency $f_i(t)$ increases during the pulse duration, according to the formula:

$$f_i(t) = f_l + (f_u - f_l)\frac{t}{T} \tag{1}$$

where f_l and f_u are the lower and the upper frequency limits, respectively, and T is the duration of the chirp pulse. In the case of the negative chirp $c_-(t)$, the instantaneous frequency $f_i(t)$ decreases during the pulse duration, accordingly:

$$f_i(t) = f - (f_u - f_l)\frac{t}{T} \tag{2}$$

Introducing a modulation index *h*, defined as for binary frequency shift keying (FSK), $h = (f_u - f_l)T = \Delta fT$, we can express $f_i(t)\,;0 < t \leq T$, as:

$$f(t) = \begin{cases} \left(f_c - \frac{h}{2T}\right) + \frac{ht}{T^2}, & \text{for} \quad c_+(t) \\ \left(f_c + \frac{h}{2T}\right) - \frac{ht}{T^2}, & \text{for} \quad c_-(t) \end{cases} \tag{3}$$

where f_c denotes the central frequency of the chirp pulse, sometimes referred to as the carrier frequency.

Hence, we can describe the waveform $c_+(t)\,;0 < t \leq T$, having an amplitude *A*, by means of:

$$c_+(t) = A\cos\left[2\pi\int_0^t f_i(\tau)d\tau + \phi_0\right] = A\cos\left[2\pi\int_0^t\left(f_c - \frac{h}{2T} + \frac{h\tau}{T^2}\right)d\tau + \phi_0\right] \quad (4)$$

which after performing an integration yields:

$$c_+(t) = A\cos\left[2\pi\left(f_c - \frac{h}{2T}\right)t + \frac{\pi h t^2}{T^2} + \phi_0\right] \quad (5)$$

By a direct analogy, the $c_-(t)$; $0 < t \le T$ waveform is given by:

$$c_-(t) = A\cos\left[2\pi\left(f_c + \frac{h}{2t}\right)t - \frac{\pi h t^2}{T^2}\phi_0\right] \quad (6)$$

In equations (4), (5), and (6), ϕ_0, being an initial phase value, is a real constant $0 \le \phi_0 < 2\pi$.

Using a bandpass signal notation, and for simplicity assuming A = 1, we can express $c_+(t)$ as:

$$c_+(t) = \begin{cases} \exp[j2\pi h q_p(t)]\exp[j(2\pi f_c t + \phi_0)], & 0 < t \le T \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

where $q_p(t)$ is an elementary phase pulse given by [9]:

$$q_p(t) = \begin{cases} \frac{t^2}{2T^2} - \frac{t}{2T}, & 0 < t \le T \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

Using the same notation, we have:

$$c_-(t) = \begin{cases} \exp[-j2\pi h q_p(t)]\exp[j(2\pi f_c t + \phi_0)], & 0 < t \le T \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

Therefore, the baseband chirp pulses are given by:

$$b_+(t) = \begin{cases} \exp[j2\pi h q_p(t)], & 0 < t \le T \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

$$b_-(t) = \begin{cases} \exp[-j2\pi h q_p(t)], & 0 < t \le T \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

Discretising the analog chirp pulse by substituting n for t, and N for T in (10), we can write a formula defining a complex polyphase chirp sequence

$$\left\{ \hat{b}_n(h) \right\} = \left(\hat{b}_n(h); \quad n = 1,2,\ldots,N \right) \tag{12}$$

where:

$$\hat{b}_n(h) = \exp[\, j2\pi h b_n\,], \quad n = 1,2,\ldots,N, \tag{13}$$

$$b_n = \frac{n^2 - 1}{2N^2} \tag{14}$$

and h can take any arbitrary nonzero real value.

Certainly, both periodic and aperiodic autocorrelation functions (ACFs) strongly depend on the value of h. To illustrate this dependency, the plots of magnitudes of aperiodic ACFs for chirp sequences are given in Figure 2 for three values of parameter h.

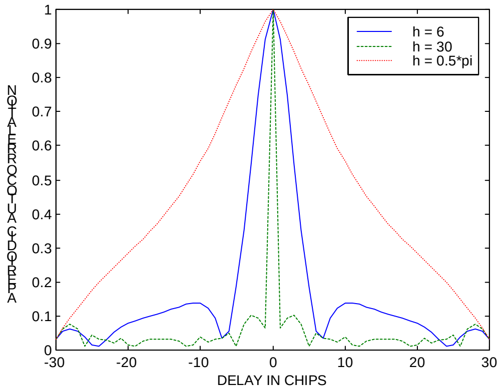

Figure 2: Magnitudes of aperiodic autocorrelation functions for example chirp sequences, $N = 31$.

The chirp sequences exhibit also good crosscorrelation properties for some pairs of parameter h values. This is illustrated in Figure 3, where the magnitude of aperiodic CCF is plotted for an example pair of chirp sequences $(h_1 = 6, h_2 = 30)$.

The main advantage of chirp sequences compared to other known sets of sequences, lies in that we can easily generate the set for any given length N. On the other hand, most of the known sets of sequences can be generated only for a certain values of N. The values of parameter h for the sequences can be optimised to achieve:

i. minimum multiaccess interference - by minimising the mean square aperiodic crosscorrelation, R_{CC} [6],

ii. the best synchronisability - by minimising the mean square aperiodic autocorrelation R_{AC} [6], *iii.* minimum peak interference - by minimising the maximum value for the aperiodic CCFs, $ACCF_{\max}(d)$, over the whole set of the sequences.

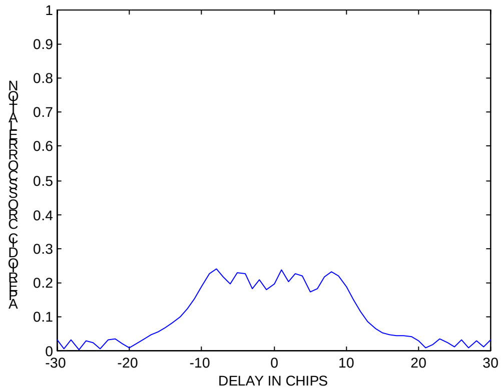

Figure 3: Magnitudes of aperiodic crosscorrelation function between a pair of chirp sequences, $N = 31, h_1 = 6, h_2 = 30$.

Usually, by improving one of the above features the other two need to be compromised. Therefore, while searching for the optimum values of parameter h for the whole set of sequences, the acceptable compromise needs to be achieved.

2.2 Example

Let us design a set of 16 polyphase chirp sequences $\{\hat{b}_n^{(r)}\}$ of length $N = 16$. Because the parameter h can take any real non-zero value, we chose them according to the following pattern[1]

$$h^{(r)} = \begin{cases} (r-9)d, & \text{for} \quad r = 1,2,...,8 \\ (r-8)d, & \text{for} \quad r = 9,10,...16 \end{cases} \tag{15}$$

To find the proper value of d, we then calculate the values of R_{CC}, R_{AC}, and $ACCF_{\max}$ for $1 \le d \le 20$ with a step of 0.1, calculated for a single sample per chip. A reasonable compromise can be reached for $d = 7.3$, with $R_{CC}(7.3) = 1.0041$, $R_{AC}(7.3) = 0.7405$, and $ACCF_{\max}(7.3) = 0.4824$. For the

1. Certainly, any other choice can be considered, and may even lead to better performance.

comparable set of 16 sequences, i.e. Gold-like sequence set of length $N = 15$, we have the following values: $R_{CC} = 0.9627$, $R_{AC} = 0.7490$, and $ACCF = 0.6000$.

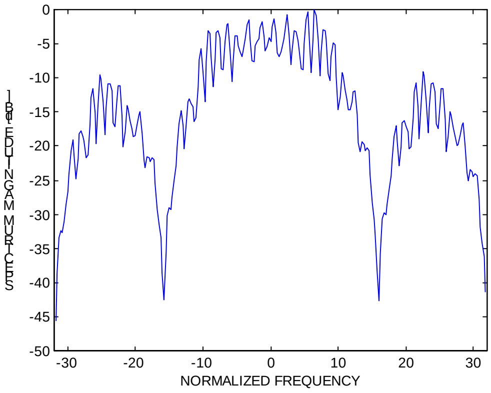

Figure 4: Power spectrum magnitude for signal obtain by spreading a random bipolar signal by the use of the chirp signature of length 16, and parameters *h* defined by the use of equation (15) with d = 7.3, r = 6.

Additional comparison of these two sets needs to be done from the viewpoint of spectral characteristics of the sequences. There are several different methods of estimating power spectrum of complex signals. We have decided to use here the nonparametric Welch method to estimate power spectrum of the spread signal. The method is implemented in a standard MATLAB Signal Processing Toolbox as a function '*psd*' [5].

To obtain a good spectral resolution, we simulated random bipolar, {+1, -1}, data sequences of length 64 bits spread by the spreading sequences, and applied sampling of four samples per chip. For other parameters of the '*psd*' function we used MATLAB default parameters [5], i.e.: FFT length - 256, Hanning window of length 256, 50% overlapping. An example plot of the obtained power spectrum is given in Figure 4.

3. SEQUENCES DESIGNED BY THE USE OF MULTIPLE CHIRPS

3.1 Design method

In the previous section, we showed that using sequences based on the baseband chirp pulses we can design useful sets of spreading sequences of any arbitrary length. Here, we will consider an extension to this idea, i.e. design of sequence sets comprising sequences designed on the basis of baseband chirp pulses of higher order or even the linear combination of them. To do so, we first introduce a definition of the chirp pulse of order *s*.

A pulse is referred to as a chirp pulse of the order *s*, if and only if the first time derivative of its instantaneous frequency (the angular acceleration) is a step function with the number of time intervals where it is constant is equal to *s*. In addition, if the integral of the instantaneous frequency over the duration of the pulse is equal to zero:

$$\int_0^T f_i(t)dt = 0 \tag{16}$$

then such a pulse is called a baseband chirp pulse of the order *s*.

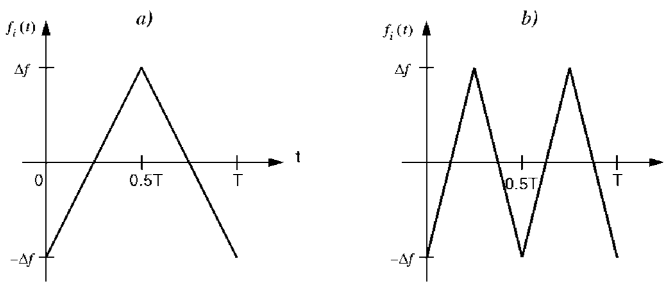

Figure 5: Example baseband chirp pulses of the order: a) 2, b) 4.

As an example, a baseband chirp pulse of orders 2 and 4 are shown in Figure 5. The presented pulses are symmetrical, in general however the chirp pulses do not need to be so regular.

The instantaneous frequency function $f_i(t)$ for the chirp pulse of order 2, depicted in Figure 5 is given by:

$$f_i(t) = \begin{cases} \frac{4t}{T}\Delta f - \Delta f, & 0 < t \le \frac{T}{2} \\ -\frac{4t}{T}\Delta f + 3\Delta f, & \frac{T}{2} < t \le T \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

Substituting h/T for Δf and performing integration gives the following expression for the elementary phase pulse $q_p(t)$ for the chirp pulse of order 2:

$$q_p(t) = \begin{cases} \dfrac{2t^2}{T^2}h - \dfrac{th}{T}, & 0 < t \le \dfrac{T}{2} \\ -\dfrac{2t^2}{T^2}h + 3\dfrac{th}{T} - 1, & \dfrac{T}{2} < t \le T \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

Substituting *n* for *t*, and *N* for *T* to discretise the pulse $q_p(t)$ given by equation (18), we obtain the formula for the elements d_n of a normalised (*h* = 1) double chirp sequence $\{d_n\}$:

$$d_n = \begin{cases} \dfrac{2n^2}{N^2} - \dfrac{n}{T}, & 0 < t \le \dfrac{N}{2} \\ -\dfrac{2n^2}{N^2} + 3\dfrac{n}{N} - 1, & \dfrac{N}{2} < t \le N \\ 0, & \text{otherwise} \end{cases} \tag{19}$$

The complex double chirp sequence elements $\hat{d}_n$ are therefore given by:

$$\hat{d}_n = \exp[j2\pi h d_n]; \quad n = 1,2,\ldots,N \tag{20}$$

In the same way, we can find the formula of the elements q_n of a normalised quadruple chirp sequence $\{q_n\}$, corresponding to the chirp pulse of the order 4. The elements q_n are expressed as:

$$q_n = \begin{cases} \dfrac{4n^2}{N^2} - \dfrac{n}{N}, & 0 < t \le \dfrac{N}{4} \\ -\dfrac{4n^2}{N^2} + 3\dfrac{n}{N} + 1, & \dfrac{N}{4} < t \le \dfrac{N}{2} \\ \dfrac{4n^2}{N^2} - 5\dfrac{n}{N} - \dfrac{3}{2}, & \dfrac{N}{2} < t \le \dfrac{3N}{4} \\ \dfrac{4n^2}{N^2} - 7\dfrac{n}{N} - 3, & \dfrac{3N}{4} < t \le N \\ 0, & \text{otherwise} \end{cases} \tag{21}$$

and the elements of the complex quadruple chirp sequences are given by:

$$\hat{q}_n = \exp[j2\pi h q_n]; \quad n = 1,2,..., N. \tag{22}$$

Analogically, one can develop the formulae describing any, even irregular, chirp sequences.

Another class of higher order chirp sequences, can be obtained if a superposition of chirp sequences of different orders is used to create the complex polyphase sequence. In the following example, we will show that by using such a superposition we can create sequence sets having better performance.

3.2 Example

Let us consider a set of 16 complex sequences $\{\hat{s}_n^{(r)}\}$, $r = 1,2,...16$, of length 16 obtained by the use of a superposition of a single and double chirp sequences. Therefore, their elements $\hat{s}_n^{(r)}(h_1^{(r)}.h_2^{(r)})$ are given by:

$$\hat{s}_n^{(r)}(h_1^{(r)}.h_2^{(r)}) = \exp[j2\pi(h_1^{(r)} b_n + \quad h_2^{(r)} d_n)]; \quad r = 1,...16, n = 1,...16, \tag{23}$$

where the coefficients $h_1^{(r)}$ and $h_2^{(r)}$ can be any real numbers, with the only exception that they cannot be equal both to zero for the same *r*.

In order to find the acceptable values for the coefficients $h_1^{(r)}$ and $h_2^{(r)}$ for all 16 sequences; let us define them in the following way:

$$h_1^{(r)} = \begin{cases} d_1(r-9), & r = 1,2,...,8 \\ d_1(r-8), & r = 9,10,...16 \end{cases} \tag{24}$$

$$h_2^{(r)} = r d_2, \quad r = 1,2,...,16, \tag{25}$$

and compute the values of $R_{CC}(d_1, d_2)$, $R_{AC}(d_1, d_2)$, and $ACCF_{\max}(d_1, d_2)$ for with a grid of 0.2.

Since the average mean aperiodic crosscorrelation is generally regarded as the most important parameter from the viewpoint of multiaccess interference in DS CDMA systems, we decided here, to chose those values of d_1 and d_2, where the minimum of $R_{CC}(d_1, d_2)$ appears, i.e. $d_1 = 14.2$, and $d_2 = 7.6$.

This results in $R_{CC}(14.2, 7.6) = 0.9057$, $R_{AC}(14.2, 7.6) = 1.7439$, and $ACCF_{\max}(14.2, 7.6) = 0.6085$. The full list of corresponding values of $h_1^{(r)}$ and $h_2^{(r)}$ for all 16 sequences is given in Table 1, and the plot of $ACCF_{\max}(\tau)$ for $d_1 = 14.2$, and $d_2 = 7.6$ is given in Figure 6.

Table 1: List of the coefficients $h_1^{(r)}$ and $h_2^{(r)}$.

r	$h_1^{(r)}$	$h_2^{(r)}$	r	$h_1^{(r)}$	$h_2^{(r)}$
1	-113.60	7.60	9	14.20	68.40
2	-99.40	15.20	10	28.40	76.00
3	-85.20	22.80	11	42.60	83.60
4	-71.00	30.40	12	56.80	91.20
5	-56.80	38.00	13	71.00	98.80
6	-42.60	45.60	14	85.20	106.40
7	-28.40	53.20	15	99.40	114.00
8	-14.20	60.80	16	113.60	121.60

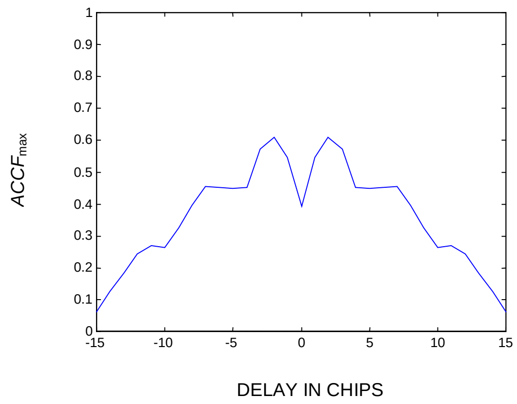

Figure 6: Plot of $ACCF_{\max}(\tau)$ for $d_1 = 14.2$, and $d_2 = 7.6$.

Another important set of characteristics for a spreading sequence set is power spectral densities of signals spread by these sequences. An example plot of the magnitude of power spectrum for signal spread by the sequence $\{\hat{s}_n^{(14)}\}$ is given in Figure 7. The plot was obtained using the same method as described in the previous section.

Further improvement in the value of R_{CC} can be obtained if coefficients $h_1^{(r)}$ and $h_2^{(r)}$ are optimised for all 16 sequences from the viewpoint of reaching minimum of R_{CC}. Several different methods of optimisation can be used for this purpose, and determining which one is the most efficient exceeds the scope of this thesis. To show however, that reduction in R_{CC} is possible, we have applied the Nelder-Meade simplex search implemented in MATLAB as the function '*fmins*' [5], choosing the values given in Table 1 as a starting point.

After completing this optimisation, we were able to achieve R_{CC} = 0.8441. However, as expected the gain was counteracted by deterioration in the synchronisability performance due to R_{AC} = 2.4875. The list of the optimised coefficients $h_1^{(r)}$ and $h_2^{(r)}$ is given in Table 2.

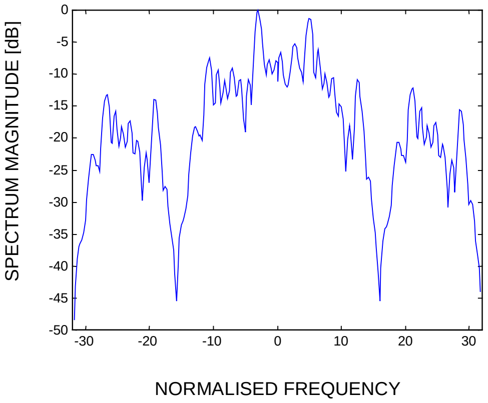

Figure 7: Plots of power spectrum magnitudes for signals obtain by spreading a random bipolar signal by the use of the sequence $\{\hat{s}_n^{(14)}\}$.

Table 2: List of the optimised coefficients $h_1^{(r)}$ and $h_2^{(r)}$.

r	$h_1^{(r)}$	$h_2^{(r)}$	r	$h_1^{(r)}$	$h_2^{(r)}$
1	-112.6513	7.5972	9	13.2249	69.7038
2	-96.7127	15.2773	10	28.2715	78.1511
3	-84.1755	23.5579	11	42.2284	85.7450
4	-71.0210	31.3700	12	57.6174	92.8630
5	-56.8045	36.4517	13	70.2810	98.7896
6	-44.9827	44.2826	14	85.4777	105.8105
7	-27.9950	49.9497	15	96.5492	112.4437
8	-14.5283	63.7458	16	114.0881	121.3066

In Figure 8 we present two example plots of the magnitude of aperiodic CCF between the sequences (1,2) and (1,16), having their parameters $h_1^{(r)}$ and $h_2^{(r)}$ according to Table 2.

Even though the value of R_{AC} = 2.4875, the aperiodic ACFs for all of the sequences are still reasonable, exhibiting significant peaks at zero. The example plots of the magnitudes of ACFs for three different sequences are given in Figure9. Certainly, one can optimise the coefficients $h_1^{(r)}$ and $h_2^{(r)}$, to minimise other parameters like R_{AC}. Also, one can chose different starting point for the optimisation.

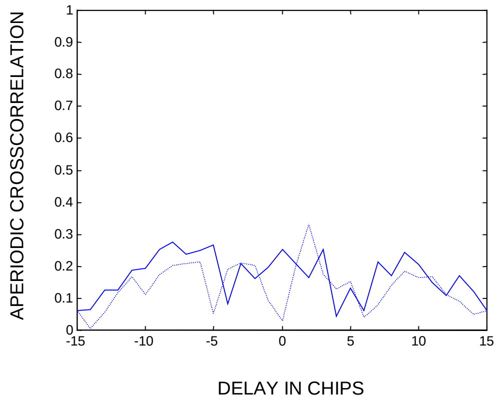

Figure 8: Magnitudes of the aperiodic CCFs between the sequences: (1,2) - solid line, and (1,16) - dotted line, having their parameters $h_1^{(r)}$ and $h_2^{(r)}$ according to Table 2.

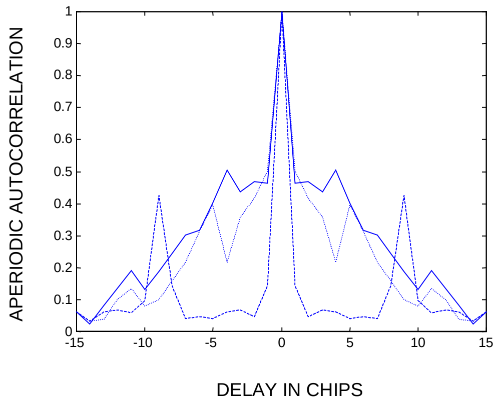

Figure 9: Magnitudes of aperiodic ACFs for r = 1, 8 and 16.

In Figure 9 we present two example plots of the magnitude of aperiodic CCF between the sequences (1,2) and (1,16), having their parameters $h_1^{(r)}$ and $h_2^{(r)}$ according to Table 2.

Even though the value of R_{AC} = 2.4875, the aperiodic ACFs for all of the sequences are still reasonable, exhibiting significant peaks at zero. The example plots of the magnitudes of ACFs for three different sequences are given in Figure10. Certainly, one can optimise the coefficients $h_1^{(r)}$ and $h_2^{(r)}$, to minimise other parameters like R_{AC}. Also, one can chose different starting point for the optimisation.

4. CONCLUSIONS

In this work we have introduced the new methods of designing polyphase spreading sequences for DS CDMA wireless networks. Both methods are based on application of discretised baseband chirp pulses or pulses containing multiple chirps or even linear combinations of them. The example sets of designed sequences exhibit good correlational properties, which is reflected in the values of mean square aperiodic crosscorrelation, R_{CC}, and mean square aperiodic autocorrelation, R_{AC}. The major benefit of the methods lies in the fact that we can design sequences of any arbitrary length, and optimise their parameters to achieve reasonable level of multiaccess interference and/or good synchronisation properties.

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