

# Performance Analysis of AMI-GAASK

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## 1 Introduction

In order to achieve the best transmission performance and the best utilisation of a radio channel, several different phase modulation schemes have been investigated for digital communications, e.g. [1,2,3,4]. Generally, they can be regarded as either Continuous Phase Modulations (CPMs) where an instantaneous value of a modulated signal phase is a continuous function of time, or modulations leading to the discontinuities in a modulated signal phase trajectories, e.g. M-PSK, M-DPSK.

The CPM signals usually have better spectral performance, as it is shown by Anderson et. al. in [4] or Couch in [5] than those with phase discontinuities, and by the use of MLSE [4] type demodulators they also achieve very good error performance. These factors are the key reasons for the growing popularity in the use and interest in a development of new kinds of CPMs [5,6,7,8]. Modulation schemes for Single Carrier Per Channel Frequency Hopping Spread Spectrum wireless LANs recently proposed before IEEE 802.11 standardisation committee [9,10,11] are also all of the CPM type.

## 2 General formulae

The proposed modulation scheme is based on setting a relationship between a modulating signal  $m(t)$  and an instantaneous value  $\varphi(t)$  of the information carrying phase component by means of a differential equation:

$$\frac{d^2}{dt^2}[\varphi(t)] = 2\pi h \cdot m(t) \quad (1)$$

Thus the  $\varphi(t)$  is expressed as:

$$\varphi(t) = 2\pi h \int_0^t \int_0^\tau m(\theta)(d\theta)d\tau \quad (2)$$

and for the isochronous sequence  $(\alpha^{(n)})$  of modulated data Formula (2) takes the form:

$$\varphi(t, (\alpha^{(n)})) = 2\pi h \sum_{i=0}^N \alpha^{(i)} \int_0^t \int_0^\tau q_a(\theta - iT)(d\theta)d\tau \quad (3)$$

where  $q_a(t)$  is an elementary modulating pulse and  $N$  is a current number of data symbol, and  $T^{-1}$  is a data rate.

To achieve a possibility of a physical realisation, i.e. a limited value of an instantaneous frequency of the modulated signal, it is necessary to ensure a non-existence of a constant component in the modulating signal  $m(t)$ . This can be done either by the use of balanced modulating pulses, i.e. pulses fulfilling the condition:

$$\int_{-\infty}^{\infty} q_a(\tau) d\tau = 0 \quad (4)$$

or by ensuring that a running digital sum,  $RDS(N)$ , of modulating data

$$RDS(N) = \sum_{i=0}^N \alpha^{(i)} + RDS(0) \quad (5)$$

is bounded.

It is simple to prove that neither unipolar nor bipolar binary uncorrelated data can satisfy the condition (5) [5]. This can be only achieved by a premodulation encoding, when a balanced line code is employed.

The class of balanced line codes is a very broad one [5], e.g. AMI, PST, 4B3T, HDB3, etc. Certainly, the modulated signal obtained as a result of different premodulation encoding has different performance. In order to choose the most suitable line code, let consider an instantaneous value of the modulated signal frequency deviation,  $\Delta f(t)$ , given by:

$$\Delta f(t) = \frac{1}{2\pi} \cdot \frac{d}{dt} \varphi(t, (\alpha^{(n)})) = h \sum_{i=0}^N \alpha^{(i)} \int_0^t q_a(\tau - iT) d\tau = h \sum_{i=0}^N \alpha^{(i)} q_f(t - iT) \quad (6)$$

where  $q_f(t)$  is an elementary frequency pulse.

From the latest formula it is visible that, apart from the scaling function of the modulation index,  $h$ , the maximum frequency deviation  $\Delta f_{max}$  depends on the shape of the elementary frequency pulse  $q_f(t)$  and on the bounds of  $RDS(N)$ . With regard to bandwidth utilisation, the line code chosen for the premodulation encoding should be characterised by minimal variations of  $RDS(N)$ . From this point of view, the optimal line code for such a task is Alternate Mark Inversion, AMI, code, characterised by the minimal deviation of  $RDS(N)$  equal to one. Later, the modulation scheme obtained by employing the AMI code is referred to as AMI-AASK (Angular Acceleration Shift Keying).

In order to achieve the best bandwidth utilisation together with the lowest error probability, the properties of the AMI-AASK signals have been analysed assuming the elementary angular acceleration pulse being gaussian pulses described by [4]:

$$q_{a-G}(t) = \frac{1}{\sqrt{\pi}} \cdot \{erfc(-\alpha t) - erfc[-\alpha(t-1)]\} \quad (7)$$

where  $\alpha$  is a real positive constant. This kind of modulation is later denoted by AMI-GAASK.

### 3 Spectral Analysis

The Power Spectral Density (PSD) of the resultant modulated signals have been estimated using the MATLAB Signal Processing Toolbox built in function 'psd' which utilises the Welch's PSD

estimation method [15,16]. To get the reliable results, the PSD estimates have been calculated for the modulated signals corresponding to random, binary sequence  $s$  of 512 bits with an accuracy of 20 samples per each time interval equivalent to a single data bit. Later, as it is commonly applied, results have been normalised, with frequency normalised according to the formula:

$$f_{norm} = T \cdot (f - f_c) \quad (8)$$

where  $f_c$  is the carrier frequency, and PSD in [dB] given by:

$$PSD(f_{norm}) = 10 \cdot \log_{10} \left[ \frac{psd}{|psd|_{f_{norm}=0}} \right] \quad (9)$$

where  $psd$  is a raw estimate returned by MATLAB "psd" function.

The example plots of PSD for the parameter  $\alpha$  equal to 1.8888 are given in Fig.1

For the comparison, in Fig.2 the plots of PSD for the Gaussian FSK with the same value of the

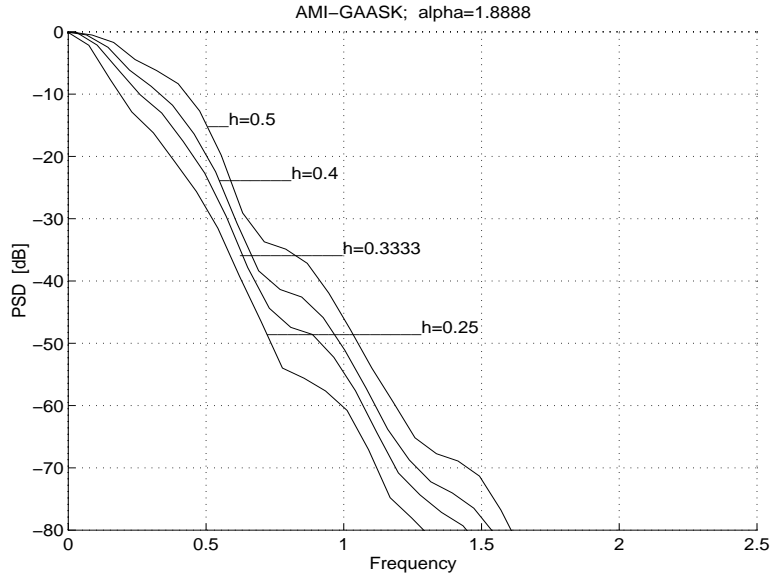


Figure 1: The plots of PSD for AMI-GAASK with the parameter  $\alpha$  equal to 1.8888.

parameter  $\alpha$  are presented. It is clearly visible that the spectral performance of AMI-GAASK are much better than those of the relevant GFSK.

## 4 Error Performance

Because AAM in general, and AMI-GAASK in particular, are the CPM modulations, the optimal detection can be achieved by the use of MLSE like demodulation [4]. For such a technique, the modulated signal parameter, being the most critical from the view point of an error performance, is a minimum Euclidean distance,  $d_{min}$ , between signals corresponding to data symbol sequences starting with a symbol "1" and sequences starting with a symbol "0", assuming the same previous bit history. In order to evaluate the  $d_{min}$ , squared Euclidean distances have been calculated between all relevant  $(2LT + 1)$  time long modulated signals, utilising the simplified formula [4]:

$$d^2(x_i(t), x_j(t)) = \frac{1}{T} \int_0^{2LT} [1 - \cos \Delta \varphi(t)] dt \quad (10)$$

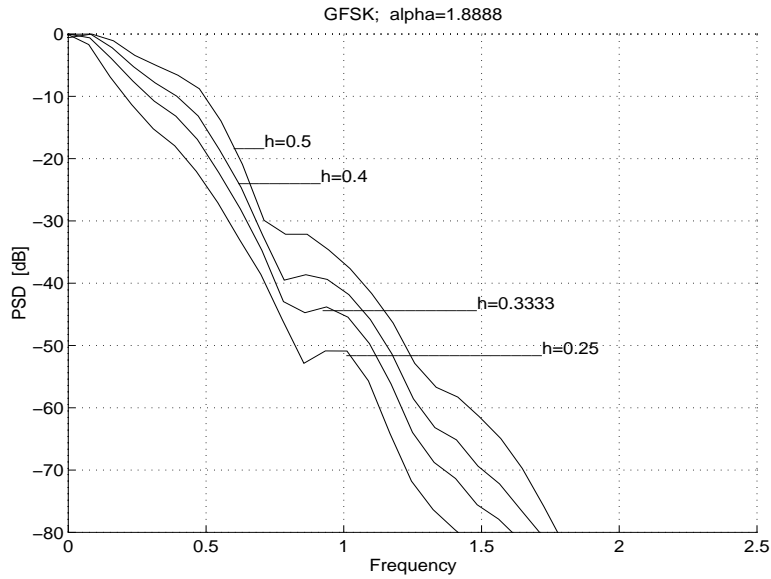


Figure 2: The plots of PSD for GFSK with the parameter  $\alpha$  equal to 1.8888.

where  $\Delta\varphi(t)$  denotes the phase difference between the modulated signals  $x_i(t)$  and  $x_j(t)$ , and  $LT$  is the duration time of the elementary angular acceleration pulse. If such a pulse is infinite, e.g. a gaussian pulse,  $LT$  is assumed to be the time interval where the pulse is significantly different from zero.

In the Table 1 the values of  $d_{min}^2$  for AMI-GAASK and relevant GFSK are compared. It is visible that the significant gain in the spectral efficiency of AMI-GAASK is partially compensated by a little degradation in the error performance. This is because of minimum Euclidean distances being slightly smaller for AMI-GAASK than for GFSK.

However, the AMI-GAASK can be a good alternative for GFSK in all cases where the spectral constraints are the most critical for the system design, as it takes place in wireless LANs.

$h$	$d_{min}^2$					
	GFSK			AMI-GAASK		
	$\alpha = 3$	$\alpha = 2.5$	$\alpha = 1.88$	$\alpha = 3$	$\alpha = 2.5$	$\alpha = 1.88$
0.5	2.24	2.23	2.18	2.18	2.16	2.10
0.4	1.72	1.68	1.60	1.60	1.57	1.49
0.3333	1.31	1.28	1.19	1.19	1.16	1.10
0.25	0.81	0.78	0.72	0.72	0.70	0.65

Table 1: Squared Minimum Euclidean Distances

## 5 Conclusion

In the paper, the new pulse format for Single Carrier Per Channel Frequency Hopping Spread Spectrum is proposed. Its performance comprises very good spectral characteristics, better than the relevant GFSK, and good immunity from WGN.

Further research will concentrate on the development of semi optimal demodulator for fading channels.

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