Chirp Sequences for Wireless Data Networks

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1. Introduction

There are several families of binary and complex spreading sequences proposed in literature [1], [2], [3], [4], [6], [7], with some of them, e.g. OV sequences [6] allowing for a good compromise between CCFs and ACFs for the whole set. There are, however, no clear ways how to chose appropriate values of parameters to achieve the desired spectral characteristics. In this paper, we propose a method to design a useful set of sequences for DS CDMA wireless data networks. Based on the fact that use of complex spreading codes introduces a phase modulation into the band-pass signal, we look into the properties of sequences obtained on the basis of a linear combination of baseband chirps [9], which are one of the analogue signals having very good autocorrelation properties. The similar approach has been used by Popovic [7] in design of his P3 and P4 sequences, utilising a single-chirp like sequences. Here, however, we will look into design of sequences for any given length *N*. Ability to do so, is very important from the viewpoint of applying such sequences in wireless data networks for variable data rates.

2. General Description

We will consider here design of sequence sets comprising sequences obtained on the basis of baseband chirp pulses of higher order or the linear combination of them. To do so, we first introduce a definition of the chirp pulse of order s.

Definition: A pulse is referred to as a chirp pulse of the order *s*, if and only if, its instantaneous frequency $f_i(t)$ increases and/or decreases linearly within the pulse, and the first time derivative of the instantaneous frequency (the angular acceleration) is a step function with the number of time intervals where it is constant is equal to *s*. In addition, if the integral of the instantaneous frequency over the duration of the pulse is equal to zero:

$$\int_T f_i(t)dt = 0 \qquad (1)$$

then such a pulse is called a baseband chirp pulse of the order *s*.

The instantaneous frequency function $f_i(t)$ for a symmetrical chirp pulse of order 2, is given by:

$$f_i(t) = \begin{cases} \frac{4t}{T}\Delta f - \Delta f, & 0 < t \le \frac{T}{2} \\ -\frac{4t}{T}\Delta f + 3\Delta f, & \frac{T}{2} < t \le T \\ 0, & \text{otherwise} \end{cases} \qquad (2)$$

Substituting *h/T* for *Δf*, yields:

$$f_i(t) = \begin{cases} \frac{4t}{T^2}h - \frac{h}{T}, & 0 < t \le \frac{T}{2} \\ -\frac{4t}{T^2}h + 3\frac{h}{T}, & \frac{T}{2} < t \le T \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

which, after integration, gives the following expression for the elementary phase pulse $q_p(t)$ for the chirp pulse of order 2:

$$q_p(t) = \begin{cases} \frac{2t^2}{T^2}h - \frac{ht}{T}, & 0 < t \le \frac{T}{2} \\ -\frac{2t^2}{T^2}h + 3\frac{ht}{T} - 1, & \frac{T}{2} < t \le T \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

From equation (4), substituting n for t, and N for T, we obtain the formula for the elements d_n of a normalised (h = 1) double chirp sequence :

$$d_n = \begin{cases} \frac{2n^2}{N^2} - \frac{n}{n}, & 0 < n \le \frac{N}{2} \\ -\frac{2n^2}{N^2} + 3\frac{n}{N} - 1, & \frac{N}{2} < n \le N \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

The complex double chirp sequence elements $\hat{d}_n$ are therefore given by:

$$\hat{d}_n = \exp[j2\pi h d_n]; \quad n = 1,2,\ldots,N \tag{6}$$

In the same way, one can develop the formulae describing any, even irregular, chirp sequences.

Another class of higher order chirp sequences can be obtained if a superposition of chirp sequences of different orders is used to create the complex polyphase sequence. In the following section, we will show that by using such a superposition we can create sequence sets having good performance.

3. Example

Let us consider a set of 16 complex sequences $\hat{s}_n^{(r)}$, $r = 1,2,\ldots,16$, of length 16 obtained by the use of a superposition of a single and double chirp sequences, having their elements $\hat{s}_n^{(r)}(h_1^{(r)}, h_2^{(r)})$ given by:

$$\hat{s}_n^{(r)}(h_1^{(r)}, h_2^{(r)}) = \exp[j2\pi(h_1^{(r)} b_n + h_2^{(r)} c_n)] \tag{7}$$

where the coefficients $h_1^{(r)}, h_2^{(r)}$ and can be any real numbers, with the only exception that they cannot be equal both to zero for the same r, and c_n is a single chirp sequence.

The values of the coefficients $h_1^{(r)}, h_2^{(r)}$ for the sequences can be optimised to achieve:

i. minimum multiaccess interference - by minimising value of an average mean square aperiodic crosscorrelation [6], [8], R_{CC},

i. the best synchronisability - by minimising value of mean square out-of-phase aperiodic autocorrelation [6], R_{AC},

ii. minimum peak interference - by minimising the maximum value for the aperiodic CCFs, ($ACCF_{max}$), over the whole set of the sequences.

In order to find the acceptable values for the coefficients and for all 16 sequences, let us define them in the following way:

$$h_1^{(r)} = \begin{cases} (r-9)d_1, & r = 1,\ldots,8 \\ (r-8)d_1, & r = 9,\ldots,16 \end{cases} \tag{8}$$

$$h_2^{(r)} = rd_2, \quad r = 1,\ldots,16 \tag{9}$$

and compute the values of R_{CC}, R_{AC}, and $ACCF_{max}$ for $1 \le d_1, d_2 \le 20$ with a grid of 0.2.

Since the average mean aperiodic crosscorrelation is generally regarded as more important from the viewpoint of multiaccess interference in DS CDMA systems, we decided here, to chose those values of d_1 and d_2, where the minimum of R_{CC} = 0.9057 appears, i.e. d_1 = 14.2, and d_2 = 7.6.

4. Conclusions

In the paper we have presented a new method to create families of polyphase sequences of any arbitrary length. The parameters of defining sequence sets, as well the constellation of multiple chirps used to specify the sequence family can be explored in order to achieve the desired properties.

In the considered example, one can optimise the coefficients, in order to minimise other parameters. Also, one can perform further optimisation utilising any conventional optimisation procedure.

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