Orthogonal Binary Sequences with Wide Range of Correlation Properties

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Abstract: In this paper, we propose a simple method for modifying Walsh-Hadamard spreading sequences to improve their correlation properties for asynchronous DS CDMA applications, while maintaining their orthogonality for perfect synchronization. Simulation results for the DS CDMA systems utilizing 32-chip modified Walsh-Hadamard sequences and are presented and compared to those achieved in the case of system utilizing pure 32-chip Walsh-Hadamard sequences.

1. Introduction

Walsh-Hadamard bipolar spreading sequences are generally used for channel separation in direct sequence code division multiple access (DS CDMA) systems, e.g. [1]. They are easy to generate, and orthogonal [2] in the case of perfect synchronization. However, the crosscorrelation between two Walsh-Hadamard sequences can rise considerably in magnitude if there is a non-zero delay shift between them. Unfortunately, this is very often the case for up-link (mobile to base station) transmission, due to the differences in the corresponding propagation delays. As a result, significant multi-access interference (MAI) [3] occurs which needs to be combated either by complicated multi-user detection algorithms [4], or a reduction in bandwidth utilization.

We propose here a simple modification to Walsh-Hadamard spreading sequences, which improves their properties in asynchronous applications. Such modified sequences are still orthogonal, but can exhibit much lower peaks in the aperiodic crosscorrelation functions and out-of-phase aperiodic autocorrelation functions. Hence, the level of MAI can be significantly reduced, if they are applied in DS CDMA systems for up-link transmission. Moreover they are characterized with much lower values of out-of-phase aperiodic autocorrelation. Hence, the use of such modified sequences can facilitate a sequence acquisition process [5]. In addition, the spectral characteristics can be much more uniform for the whole set of the modified sequences than for the original set of Walsh-Hadamard sequences, allowing for more uniform spreading among different channels.

2. Modification Method

The Walsh-Hadamard sequences of the length N; $N = 2^n$, $n = 1,2, \ldots$, are often defined using Hadamard matrices $\mathbf{H}_N$ [2], with

$$\mathbf{H}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{1}$$

and

$$\mathbf{H}_{2N} = \begin{bmatrix} \mathbf{H}_N & \mathbf{H}_N \\ \mathbf{H}_N & -\mathbf{H}_N \end{bmatrix}. \tag{2}$$

The resulting matrices $\mathbf{H}_N$ are orthogonal matrices, i.e. for every N we have:

$$\mathbf{H}_N \mathbf{H}_N^T = N\mathbf{I}_N \tag{3}$$

where $\mathbf{H}_N^T$ is the transposed Hadamard matrix of order N, and $\mathbf{I}_N$ is the $N \times N$ unity matrix. The modification proposed here is achieved by taking another orthogonal $N \times N$ matrix $\mathbf{D}_N$, and the new set of sequences is based on a matrix $\mathbf{W}_N$, given by:

$$\mathbf{W}_N = \mathbf{H}_N \mathbf{D}_N . \tag{4}$$

The matrix $\mathbf{W}_N$ is also orthogonal, since:

$$\mathbf{W}_N \mathbf{W}_N^T = \mathbf{H}_N \mathbf{D}_N (\mathbf{H}_N \mathbf{D}_N)^T = \mathbf{H}_N \mathbf{D}_N \mathbf{D}_N^T \mathbf{H}_N^T \tag{5}$$

and because of the orthogonality of matrix $\mathbf{D}_N$, we have

$$\mathbf{D}_N \mathbf{D}_N^T = k\mathbf{I}_N \tag{6}$$

where k is a real constant. Substituting (6) into (5) yields

$$\mathbf{W}_N \mathbf{W}_N^T = k\mathbf{H}_N \mathbf{H}_N^T = kN\mathbf{I}_N \ . \tag{7}$$

In addition, if $k = 1$, then the sequences defined by the matrix $\mathbf{W}_N$ are not only orthogonal, but possess the same normalization as the Walsh-Hadamard sequences. However, other correlation properties of the sequences defined by $\mathbf{W}_N$ can be significantly different to those of the original Walsh-Hadamard sequences.

From equation (4) it is not clear how to chose the matrix $\mathbf{D}_N$ to achieve the desired properties of the sequences defined by the $\mathbf{W}_N$. In addition, there are only a few known methods to construct the orthogonal matrices, such as those used for the Hadamard matrices themselves. However, another simple class of orthogonal matrices are diagonal matrices with their elements $d_{i,j}$ fulfilling the condition:

$$\left|d_{m,n}\right| = \begin{cases} 0 & \text{for} \quad m \neq n \\ k & \text{for} \quad m = n \end{cases}; \quad m, n = 1, \ldots, N \tag{9}$$

To preserve the normalization of the sequences, the elements of $\mathbf{D}_N$, being in general complex numbers, must be of the form:

$$d_{m,n} = \begin{cases} 0 & \text{for} \quad m \neq n \\ \exp(j\phi_m) & \text{for} \quad m = n \end{cases}; \quad m, n = 1, \ldots, N \tag{10}$$

where the phase coefficients ϕ_m; m = 1, 2, …, N, are real numbers taking their values from the interval [0, 2π), and $j^2 = -1$. The values of ϕ_m; m = 1, 2, …, N, can be optimized to achieve the desired correlation and/or spectral properties, e.g. minimum out-off-phase autocorrelation or minimal value of peaks in aperiodic crosscorrelation functions.

2. Numerical Example

From the implementation point of view, the most important class of spreading sequences are bipolar or biphase sequences, where the ϕ_m; m = 1, 2, …, N, can take only two values 0 and π. This results in the elements on the diagonal of $\mathbf{D}_N$ being equal to either '+1' or '-1'. Even for this bipolar case, we can achieve significantly different properties of the sequences defined by the $\mathbf{W}_N$ than those of the original Walsh-Hadamard sequences of the same length.

As an example, let us compare some properties of Walsh-Hadamard sequences, with the properties of the sequence set defined by the bipolar matrix $\mathbf{W}_{32}$, with the diagonal of the $\mathbf{D}_{32}$ represented for the simplicity by a sequence of '+' and '-' corresponding to '+1' and '-1', respectively:

{++++---+++--+-+-+--++-+---+----+} (11)

For the unmodified set of 32 Walsh-Hadamard sequences of length N = 32, the maximum in the aperiodic crosscorrelation function [6] (ACC_{max}) reaches 0.9688, and the mean square out-of-phase aperiodic autocorrelation [6] ($MSAAC$) is equal to 6.5938. That high value of $MSAAC$ indicates the possibility of significant difficulties in the sequence acquisition process, and the high value of ACC_{max} means that for some time shifts the interference between the different DS CDMA channels can be unacceptably high. On the other hand, for the set of sequences defined by the matrix $\mathbf{W}_{32}$ considered here, we have ACC_{max} = 0.4375, and $MSAAC$ = 0.8438. This means lower peaks in the instantaneous bit-error-rate due to the MAI and a significant improvement in the sequence acquisition process.

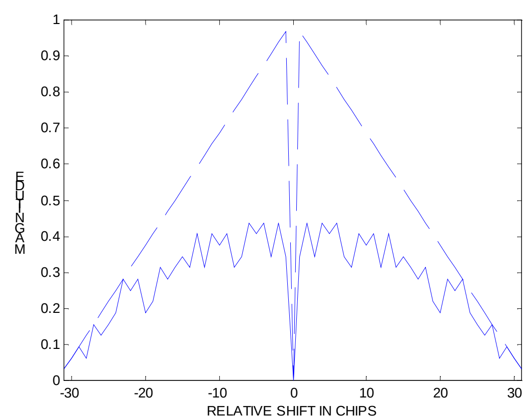

Figure 1: Plots of the maximum value of the aperiodic crosscorrelation ACC_{max} for all possible pairs of the sequences versus the relative shift between them; 32-chip modified Walsh-Hadamard sequences - solid line, 32-chip pure Walsh-Hadamard sequences – dashed line.

In Figure 1, we present the plot of the upper limits for the aperiodic crosscorrelation functions for the set of Walsh-Hadamard sequences and the set of sequences defined by our matrix $\mathbf{W}_{32}$. The presented plots illustrate the improvement that can be achieved by our modification. This, of course, translates directly on the level of BER caused by MAI. In Figure 2 and Figure 3 we present the simulation results for the 32 channel asynchronous DS CDMA system utilizing pure Walsh-Hadamard sequences and the sequences defined by our matrix $\mathbf{W}_{32}$, respectively. In both cases, we had simulated the same number of randomly chosen active users interfering with the same channel, and transmitted the same number of 524-bit frames. It can be seen that not only the average BER drops in the case of our sequences, but what is even more significant, the number of errors in the frames drops by 67%.

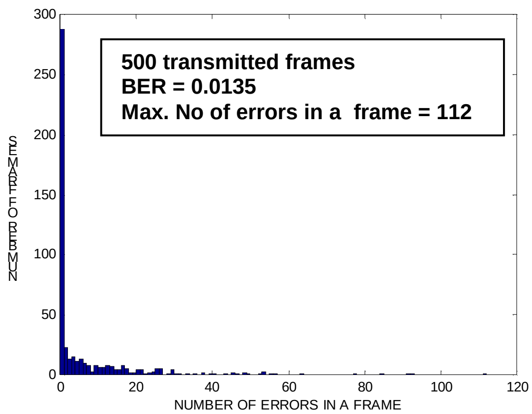

Figure 2: Histogram of a number of errors in a transmitted frame for the DS CDMA system utilizing Walsh-Hadamard spreading sequences.

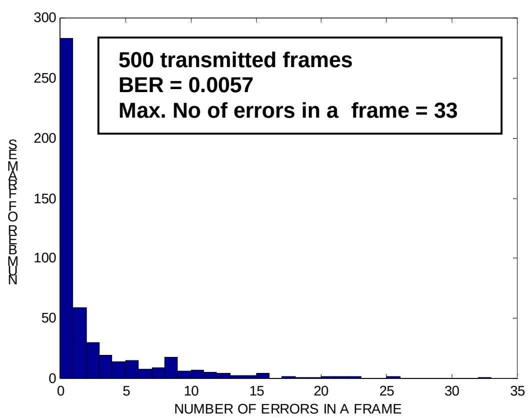

Figure 3: Histogram of a number of errors in a transmitted frame for the DS CDMA system utilizing spreading sequences defined by the matrix $\mathbf{W}_{32}$.

3. Conclusions

In the paper we presented a simple method for modifying the Walsh-Hadamard spreading sequences to improve their correlation properties for asynchronous applications, while maintaining their orthogonality for perfect synchronization. The method leads, in general, to the complex polyphase sequences but can also be used to obtain real bipolar sequences. In the case of polyphase sequences, the phase coefficients can be optimized to achieve the required correlation/spectral properties of the whole set of sequences. The presented example shows that for practical applications, where bipolar sequences are preferred, the method can also produce a significant improvement in the properties of the sequence set over those of pure Walsh-Hadamard sequences. For the asynchronous DS CDMA system utilizing our sequences has lower BER and significantly smaller number of errors per frame than can be achieved when the system utilizes pure Walsh-Hadamard sequences.

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