PERFORMANCE OF A HYBRID ARQ SCHEME FOR PARTIALLY RELIABLE TRANSPORT SERVICES

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ABSTRACT - *In [1], it was shown that provision of reliable transport services in networks which experience loss of packets can be very costly and hence accepting losses up to a certain level might be considered as an alternative approach. Recently, a family of generalised weighted sum codes (GWSC) for erasure correction was discovered [2,3] which are attractive candidates to support such a partially reliable transmission. In this paper, we present a new partially reliable hybrid ARQ scheme based on GWSC along with performance estimates. The idea can be used with some other non-binary codes too, for example Reed-Solomon codes.*

1. INTRODUCTION

In most communication networks, information digits are organised in units of fixed length, so-called packets. Because of channel impairments a transmitted packet can be received correctly, in error or might be completely lost. For example, in ATM networks the first and last cases are prevalent. Error control codes can be used to correct errors within a packet (local protection). A major objective when designing an error control scheme for a particular application is to keep redundancy low while providing throughput as high as possible. Therefore, adaptive error control codes are often employed for local protection, especially when a channel is non-stationary [4,5]. If errors are such severe that whole packets are lost, it is promising to protect sequences of packets by means of overhead packets (global protection) which then contain the redundancy [6-8]. However, some kind of adaptive global protection is still desirable. Since provision of reliable transport services in networks which experience packet losses can be very costly, losses up to a certain level might be considered as alternative approach. In that case, we call the transport service as being partially reliable.

In this paper, we propose a hybrid ARQ scheme for partially reliable transport services. The scheme is based on generalised weighted sum codes (GWSC) which were introduced in [2,3] and are used here for erasure correction and global protection. GWSC offer efficient software and hardware implementation as required in high-speed networks. Because of specific code properties in terms of weight distribution, we consider GWSC with a capability of correcting up to three erasures. An advantage of the scheme is that redundant packets have to be transmitted only if a packet has been actually lost or detected being in error. This makes it attractive for wireless local area networks (LANs) as well as for high speed wired LANs. Note that other non-binary block codes could be used instead of GWSC as well, e.g. Reed-Solomon codes.

The paper is organised as follows. A basic background on GWSC is provided in Section 2. In Section 3, the general description of the proposed ARQ/FEC scheme is presented and some of its characteristics are discussed. Section 4 presents expressions to estimate the throughput performance of the hybrid ARQ scheme. Numerical results are contained in Section 5 and provide insights into performance of the hybrid ARQ scheme. Section 6 concludes the paper.

2. GENERALISED WEIGHTED SUM CODES FOR THREE ERASURE CORRECTION

We first review some fundamentals of Weighted Sum Codes (WSC) [9], their application for single error symbol correction, and then introduce GWSC for three erasure correction.

Let k and h denote the number of data and parity bits, respectively, giving a $k+h$ bit long codeword. A WSC uses polynomial arithmetic and organises sequences of bits into sequences of symbols $h/2$ bits long. A WSC encoder processes a sequence of $n \le 2k/h$ information symbols Q_i and concatenates parity symbols P_1 and P_0. Parities are computed as follows

$$P_1 = \sum_{i=0}^{n-1} W_i \otimes Q_i \mod M$$

$$P_0 = P_1 \oplus \sum_{i=0}^{n-1} Q_i \mod M$$

where $\otimes$ and $\oplus$, respectively, denote multiplication and addition modulo a primitive polynomial M of degree $h/2$. Symbols Q_i, weights W_i, and parities P_i are elements of the Galois field $F = GF(2^{h/2})$ where M can be used to generate all non-zero elements of F. In general, weights W_i are determined by the particular coding rule but should go through a maximum length sequence to give maximum length of codewords. As far as combination of error detection capabilities and implementation complexity are concerned, WSC codes are shown to outperform CRC codes, Fletcher checksum, Internet checksum, and XTP CXOR.

In [2], it was recognised that WSC are able to correct a single symbol error as well and are equivalent to lengthened Reed-Solomon codes over $GF(2^{h/2})$ defined by a parity check matrix

$$\mathbf{H} = \begin{bmatrix} \alpha^{2^{h/2}-2} & \alpha^{2^{h/2}-3} & \dots & \alpha & 1 & 0 \\ 1 & 1 & \dots & 1 & 1 & 1 \end{bmatrix}$$

Realisation of a corresponding decoder is based on a slightly modified algorithm compared to the original one for error detection but still keeps the attractive implementation properties. Especially, a WSC decoder processes $h/2$ bits per clock cycle instead of a single bit. Syndromes can be easily computed by [2]

$$\begin{aligned} S_1 &= \hat{P}_1 \oplus \tilde{P}_1 \\ S_0 &= \hat{P}_0 \oplus \tilde{P}_0 \end{aligned}$$

where in both equations the respective received and recalculated parities are added to give two syndrome symbols. In case when syndromes indicate that error correction is required, we can determine the error position i through [2]

$$W_i = S_1 \otimes S_0^{-1}$$

It is natural to ask for a WSC family with increased correcting capabilities. In this paper, we propose to restrict ourselves to erasure correction. Thus positions of unreliable symbols are known and computation of weights W_i from syndromes S_1 and S_0 can be avoided. The GWSC used for three-erasure correction can be described using following matrix [3]:

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & \dots & 1 & 1 & 0 & 0 \\ \alpha^{q-2} & \alpha^{q-3} & \dots & \alpha & 0 & 1 & 0 \\ \alpha^{2(q-2)} & \alpha^{2(q-3)} & \dots & \alpha^2 & 0 & 0 & 1 \end{bmatrix}$$

3. A HYBRID ARQ SCHEME FOR PARTIALLY RELIABLE TRANSPORT SERVICES

The hybrid ARQ scheme uses a three erasure-correcting GWSC of length $n = k + 3$ for global protection. The k information symbols Q_i are followed by three parity symbols P_i and incorporate local protection or sequence numbering information used to detected errors or losses, respectively.

Principal operation and fundamental design parameters of the protocol are illustrated in the timing diagram Figure 1. First of all, time slots consist of k symbols and this specifies a k-boundary. In first instance, only information symbols $Q_i^{(\nu)}$, $i = 0,1,\dots,k-1$ of the ν-th information sequence are sent whereas parity symbols $P_i^{(\nu)}$, $i = 0,1,2$ are stored at the transmitter. Because of round-trip delay, we introduce a g-boundary, after which parity symbols may be released if requested. Additionally, we assume that the length k of an information sequence is larger than round-trip delay N, i.e. $N \le k$ where N denotes round-trip delay in symbols. The parameter g is chosen in the range $N \le g \le k$. The value of g can be chosen to accommodate the parity symbols associated with the actual information sequence and a predefined number of retransmitted parity symbols associated with older information sequences. Further, we assume that the receiver is able to control sequence numbers of received blocks and to distinguish between redundancy and information sequences. In other words, the receiver is synchronised with the transmitter and is able to detect the k-boundary and g-boundary.

Operation of transmitter and receiver is shown in the Table I and Table II.

TABLE I

Number of erasures in an information sequence is zero or exceeds three.

Transmitter
• Send symbols Q_i, $i = 1,\ldots,k$.
• Compute parity symbols P_0, P_1, P_2.
• Count NACK from receiver.
• Clear buffer of parity symbols if number of NACK for information symbols Q_i, $i = 1,\ldots,k$ exceeds three.
Receiver
• Check local correctness and/or sequence number of received symbols $\hat{Q}_i$, $i = 1,\ldots,k$ and send ACK or NACK to transmitter.
• Count NACK sent to transmitter.
• Release $\hat{Q}_i$, $i = 1,\ldots,k$ after k-boundary.

TABLE II

Number of erasures in an information sequence is greater than zero but less or equal to three.

Transmitter
• Send symbols Q_i, $i = 1,\ldots,k$ of the new codeword until k-boundary is reached.
• Transmit as many parity symbols P_ε as erasures ε or lost symbols have been detected.
• Retransmit parity symbols associated with older information sequences until the g-boundary is reached.
• Information sequences are declared as lost if the associated parity symbols cannot be retransmitted within the g-boundary or number of requested retransmissions exceeds a defined threshold.
• Clear buffer of parity symbols associated with lost information sequences.
Receiver
• As long as no parity symbols are received, it operates according to Table I.
• After receiving parity symbols, it determines to which information sequence the parities belong.
• Checks whether an information sequence has to be declared as finally being lost or detected in error.
• Calculate syndromes for an information sequence and correct erasures.
• Release corrected $\hat{Q}_i$, $i = 1,\ldots,k$ after k-boundary and discard losses.

Let us stress here some general characteristics of the introduced hybrid ARQ scheme:

a) With respect to Figure 1, we can recognise two windows, a *g*-window of length *g* and a *k*-window of length *k*. These windows are terminated correspondingly at g-boundary and k-boundary. As long as no erasures or losses occur, the g-window is fully embedded in the *k*-window. Otherwise, parity symbols have to be transmitted and are allocated in the first section of the *g*-window followed by information symbols. In the limiting case, there is no overlapping between *g*-window and *k*-window.

b) The value of parameter *g* basically controls the complexity required for scheduling tasks and the occurrence of a lost packet due to the operation of this hybrid ARQ scheme.

c) Note that the code used for local protection only needs to detect an error and hence might be chosen as a simple error detection code.

d) As long as no erasures or losses occur, redundancy is kept at the transmitter and consumes no transmission time. In this mode of operation, only the redundancy of the simple code used for local protection reduces slightly system throughput.

4. THROUGHPUT PERFORMANCE

To provide a preliminary throughput estimate of the proposed hybrid ARQ scheme, we assume that it is used on a binary symmetric channel (BSC). This approach is taken to simplify analysis and to illustrate fundamental behavior of the scheme in comparison to selective repeat ARQ (SR ARQ).

Local protection is given by a simple error control code of length n and code rate R. Performance characteristics of the code used for local protection are:

P_c : Probability of correct decoding,
P_f : Probability of error.

Later on SR ARQ is used as a benchmark scheme and its throughput is given by

$$\eta_{SR} = R \cdot P_c$$

In order to derive an equation for throughput performance of the proposed hybrid ARQ scheme for partially reliable transport services, we define the number of permitted retransmissions as one. Then, throughput can be computed as

$$\eta_{HARQ} = R \cdot \left[I(0) + I(1) + I(2) + I(3) + I(4\, to\, k) \right]$$

where contributions of packets with zero to three erasures are given by

$$I(0) = P_c^k$$

$$I(1) = \binom{k}{1} P_f P_c^{k-1} \left[\frac{k}{k+1} P_c + \frac{k-1}{k+1} P_f \right]$$

$$I(2) = \binom{k}{2} P_f^2 P_c^{k-2} \left[\frac{k}{k+2} P_c^2 + \frac{k-1}{k+2} 2P_f P_c + \frac{k-1}{k+2} P_f^2 \right]$$

$$I(3) = \binom{k}{3} P_f^3 P_c^{k-3} \left[\frac{k}{k+3} P_c^3 + \frac{k-1}{k+3} 3P_f P_c^2 + \frac{k-2}{k+3} 3P_f^2 P_c^1 + \frac{k-1}{k+3} P_f^3 \right]$$

and the additional term which comprises throughput contribution of those packets with more than three erasures is given by

$$I(4\ to\ k) = \sum_{i=4}^{k} \left(\frac{k-i}{k} \right) \cdot \binom{k}{i} P_f^i\ P_c^{k-1}$$

Because η_{SR} and η_{HARQ} both contain code rate R as a factor, we can use normalised throughput

$$\eta'_{SR} = \frac{\eta_{SR}}{R} \quad \text{and} \quad \eta'_{HARQ} = \frac{\eta_{HARQ}}{R}$$

5. Numerical Results

Let us compare throughput of standard SR ARQ with the proposed hybrid ARQ scheme. SR ARQ is used for blocks of length n=424 (ATM cell) and local protection is based on a simple 16bit cyclic redundancy check. On the other hand, the hybrid ARQ scheme considers a block of 424 bits as a single symbol where k symbols represent an information word. Both schemes are use on a BSC.

Figure 2 shows normalised throughput versus bit error rate for the special case when omitting $I(4\ to\ k)$ in η_{HARQ}. This gives an indication on the lower bound of throughput. It also shows that considering blocks of length k with more than three erasures as being lost does not effect throughput as severe as one would expect. This is thought to be caused by the declining probability of multiple symbol errors in a block of length k.

Figure 3 shows normalised throughput including contributions of those packets that contain more than three erasures. Obviously, the proposed ARQ scheme can provide the same throughput as SR ARQ and even outperforms SR ARQ when packet length is $k=3$. However, a certain amount of lost symbols in a packet has to be accepted.

6. Conclusion

There is a huge amount of combinations how this simple scheme can be adapted for different practical situations. The presented discussion only pretends to give a basic description and a preliminary throughput estimate of the scheme. In practice the great degree of freedom allows the designers of specific application and related hardware and/or software components to choose an optimal trade off between reliability and complexity. A thorough throughput analysis of the presented hybrid ARQ scheme is in progress and results will be published elsewhere.

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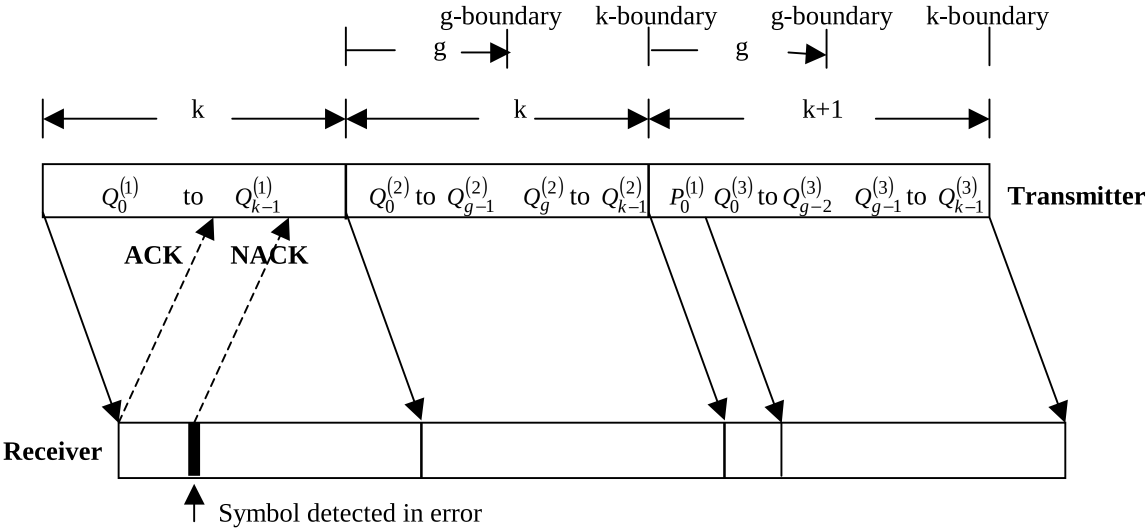

Fig.1: Example of a timing diagram for the hybrid ARQ scheme with a single error detected in the first transmitted information sequence (ACK=positive acknowledgment, NACK=negative acknowledgment)

Fig.2: Normalised throughput for SR ARQ and proposed hybrid ARQ (HARQ) without inclusion of $I(4\ to\ k)$ (Binary symmetric channel, n=424).

Fig.3: Normalised throughput for SR ARQ and proposed hybrid ARQ (HARQ) with inclusion of $I(4\ to\ k)$ (Binary symmetric channel, n=424).