# Modified Walsh-Hadamard Sequences for DS CDMA Wireless Systems

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**Abstract:** We propose a simple but efficient method for modifying Walsh-Hadamard sequences to achieve correlation properties suited for asynchronous DS CDMA applications. The proposed method can be used to minimize the mean square value of aperiodic cross-correlation or the mean square value of aperiodic autocorrelation, the maximum value of aperiodic cross-correlation functions, merit factor or other properties of the sequence set. The important feature of the method is that it modifies correlation properties of the sequence set, while preserving their orthogonality for the perfect synchronization. The proposed method can be applied to obtain bipolar, quadri-phase, or general polyphase sequences.

**Keywords:** spreading sequences, orthogonal sequences, code division multiple access, correlation functions

# 1. Introduction

Walsh-Hadamard bipolar spreading sequences can be used for channel separation in direct sequence code division multiple access (DS CDMA) systems, e.g. [1]. They are easy to generate, and orthogonal [2] in the case of perfect synchronization. However, the cross-correlation between two Walsh-Hadamard sequences can rise considerably in magnitude if there is a non-zero delay shift between them. Unfortunately, this is very often the case for up-link (mobile to base station) transmission, due to the differences in the corresponding propagation delays. As a result, significant multi-access interference (MAI) [3] occurs which needs to be combated either by complicated multi-user detection algorithms [4], or reduction in bandwidth utilization. Moreover, due to their very regular structure, Walsh-Hadamard sequences are characterized with very poor auto-correlation properties. In real systems, this is alleviated by the use of scrambling codes on the top of Walsh-Hadamard sequences. These are normally very long codes having very distinctive peaks at zero in their auto-correlation functions. For example, in UMTS these cods are $2^{18}$ bits long [5]. In addition to improving synchronization properties, scrambling also helps in reducing MAI.

Another possible solution to this problem can be use of orthogonal polyphase spreading sequences, like those proposed in [6], which for some values of their parameters can exhibit a reasonable compromise between autocorrelation and cross-correlation functions. However, in most cases, the choice of the parameters is not simple. In addition, improving one of the characteristics is usually associated with a significant worsening of the others [6]. Polyphase spreading sequences are rather difficult to implement as this require an analog phase modulator.

In [7], we proposed a method to optimize orthogonal polyphase spreading sequences for wireless data applications. That method can be also applied to modify correlation characteristics of bipolar or quadri-phase sequences. In this paper, we present an application of that method to modify Walsh-Hadamard spreading sequences in order to improve their properties in asynchronous applications. Such modified sequences are still orthogonal, but can exhibit much lower peaks in the aperiodic cross-correlation functions. Hence, the level of MAI can be significantly reduced, if they are applied in DS CDMA systems for up-link transmission. Moreover they are characterized with much lower values of out-of-phase aperiodic auto-correlation. Thus, the use of such modified sequences can facilitate a sequence acquisition process [4]. In addition, the spectral characteristics can be much more uniform for the whole set of the modified sequences than for the original set of Walsh-Hadamard sequences, allowing for more uniform spreading among different channels.

The paper is organized as follows. In Section 2, we introduce the method used later to modify correlation characteristics of Walsh-Hadamard sequences. Section 3 introduces the correlation measures, which can be considered while modifying Walsh-Hadamard sequences for DS CDMA applications. The numerical examples of modification resulting in the sets of bipolar and quadri-phase sequence sets are given in Section 4 and Section 5, respectively, and Section 6 concludes the paper.

## 2. Modification Method

The Walsh-Hadamard sequences of the length $N$; $N = 2^n$, $n = 1,2, \ldots$, are often defined using Hadamard matrices $\mathbf{H}_N$ [2], with

$$\mathbf{H}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{1}$$

and

$$\mathbf{H}_{2N} = \begin{bmatrix} \mathbf{H}_N & \mathbf{H}_N \\ \mathbf{H}_N & -\mathbf{H}_N \end{bmatrix}. \tag{2}$$

The resulting matrices $\mathbf{H}_N$ are orthogonal matrices, i.e. for every $N$ we have:

$$\mathbf{H}_N \mathbf{H}_N^T = N\mathbf{I}_N \tag{3}$$

where $\mathbf{H}_N^T$ is the transposed Hadamard matrix of order $N$, and $\mathbf{I}_N$ is the $N \times N$ unity matrix. The modification proposed here is achieved by taking another orthogonal $N \times N$ matrix $\mathbf{D}_N$, and the new set of sequences is based on a matrix $\mathbf{W}_N$, given by:

$$\mathbf{W}_N = \mathbf{H}_N \mathbf{D}_N \tag{4}$$

The matrix $\mathbf{W}_N$ is also orthogonal, since:

$$\mathbf{W}_N \mathbf{W}_N^T = \mathbf{H}_N \mathbf{D}_N (\mathbf{H}_N \mathbf{D}_N)^T = \mathbf{H}_N \mathbf{D}_N \mathbf{D}_N^T \mathbf{H}_N^T \tag{5}$$

and because of the orthogonality of matrix $\mathbf{D}_N$, we have

$$\mathbf{D}_N \mathbf{D}_N^T = k\mathbf{I}_N \tag{6}$$

where $k$ is a real constant. Substituting (6) into (5) yields

$$\mathbf{W}_N \mathbf{W}_N^T = k\mathbf{H}_N \mathbf{H}_N^T = kN\mathbf{I}_N\,. \tag{7}$$

In addition, if $k$ = 1, then the sequences defined by the matrix $\mathbf{W}_N$ are not only orthogonal, but possess the same normalization as the Walsh-Hadamard sequences. However, other correlation properties of the sequences defined by $\mathbf{W}_N$ can be significantly different to those of the original Walsh-Hadamard sequences.

From equation (4) it is not clear how to chose the matrix $\mathbf{D}_N$ to achieve the desired properties of the sequences defined by the $\mathbf{W}_N$. In addition, there are only a few known methods to construct the orthogonal matrices, such as those used for the Hadamard matrices themselves. However, another simple class of orthogonal matrices are diagonal matrices with their elements $d_{i,j}$ fulfilling the condition:

$$\left|d_{m,n}\right| = \begin{cases} 0 & \text{for} \quad m \neq n \\ k & \text{for} \quad m = n \end{cases}; \quad m, n = 1, \ldots, N \tag{9}$$

To preserve the normalization of the sequences, the elements of $\mathbf{D}_N$, being in general complex numbers[1], must be of the form:

$$d_{m,n} = \begin{cases} 0 & \text{for} \quad m \neq n \\ \exp(j\phi_m) & \text{for} \quad m = n \end{cases}; \quad m, n = 1, \ldots, N \tag{10}$$

where the phase coefficients $\phi_m$; $m$ = 1, 2, …, $N$, are real numbers taking their values from the interval [0, 2π), and $j^2 = -1$. The values of $\phi_m$; $m$ = 1, 2, …, $N$, can be optimized to achieve the desired correlation and/or spectral properties, e.g. minimum out-off-phase autocorrelation or minimal value of peaks in aperiodic cross-correlation functions.

## 3. Correlation Measures

In order to compare different sets of spreading sequences, we need a quantitative measure for the judgment. Therefore, we introduce here some useful criteria, which can be used for such a purpose. They are based on correlation functions of the set of sequences, since both the level of multiaccess interference and synchronization

[1] When elements of the matrix $\mathbf{D}_N$ are complex numbers, the simple transposition $(\bullet)^T$ must be substituted by a Hermitian transposition $(\bullet)^H$, i.e. transposition and taking complex conjugate of the elements of $\mathbf{D}_N$.

amiability depend on the cross-correlations between the sequences and the autocorrelation functions of the sequences, respectively. There are, however, several specific correlation functions that can be used to characterize a given set of the spreading sequences [3], [8], [12].

In 1969, Anderson and Wintz [13] published one of the first detailed investigations of the asynchronous DS CDMA system performance. They obtained a bound on the signal-to-noise ratio at the output of the correlation receiver for a CDMA system with hard-limiter in the channel. They also clearly demonstrated in their paper the need for considering the aperiodic cross-correlation properties of the spreading sequences. Since that time, many additional results have been obtained (e.g. [3] and [14]), which helped to clarify the role of aperiodic correlation in asynchronous DS CDMA systems.

For general polyphase sequences $\{s_n^{(i)}\}$ and $\{s_n^{(l)}\}$ of length $N$, the discrete aperiodic correlation function is defined as [12]:

$$c_{i,k}(\tau) = \begin{cases} \frac{1}{N} \sum_{n=0}^{N-1-\tau} s_n^{(i)} [s_{n+\tau}^{(l)}]^*, & 0 \le \tau \le N-1 \\ \frac{1}{N} \sum_{n=0}^{N-1+\tau} s_{n-\tau}^{(i)} [s_n^{(l)}]^*, & 1-N \le \tau < 0 \\ 0, & |\tau| \ge N \end{cases} \qquad (11)$$

where $[\bullet]^*$ denotes a complex conjugate operation. When $\{s_n^{(i)}\} = \{s_n^{(l)}\}$, Eqn. (11) defines the discrete aperiodic auto-correlation function.

Another important parameter used to assess the synchronization amiability of the spreading sequence $\{s_n^{(i)}\}$ is a merit factor, or a figure of merit [15], which specifies the ratio of the energy of autocorrelation function main-lobes to the energy of the auto-correlation function side-lobes in the form:

$$F = \frac{c_i(0)}{2\sum_{\tau=1}^{N-1} |c_i(\tau)|^2} \tag{12}$$

In DS CDMA systems, we want to have the maximum values of aperiodic cross-correlation functions and the maximum values of out-of-phase aperiodic autocorrelation functions as small as possible, while the merit factor as great as possible for all of the sequences used.

The bit error rate (BER) in a multiple access environment depends on the modulation technique used, demodulation algorithm, and the signal-to-noise power ratio (SNR) available at the receiver. Pursley [3] showed that in case of a BPSK asynchronous DS CDMA system, it is possible to express the average SNR at the receiver output of a correlator receiver of the *i*th user as a function of the average interference parameter (AIP) for the other *K* users of the system, and the power of white Gaussian noise present in the channel. The SNR for *i*th user, denoted as $\mathrm{SNR}_i$, can be expressed in the form:

$$\mathrm{SNR}_i = \left( \frac{N_0}{2E_b} + \frac{1}{6N^3} \sum_{\substack{k=1 \\ k \neq i}}^{K} \rho_{k,i} \right)^{-0.5} \tag{13}$$

where $E_b$ is the bit energy, $N_0$ is the one-sided Gaussian noise power spectral density, and $\rho_{k,i}$ is the AIP, defined for a pair of sequences as

$$\rho_{k,i} = 2\mu_{k,i}(0) + \mathrm{Re}\{\mu_{k,i}(1)\} \tag{14}$$

The cross-correlation parameters $\mu_{k,i}(\tau)$ are defined by:

$$\mu_{k,i} = N^2 \sum_{n=1-N}^{N-1} c_{k,i}(n)[c_{k,i}(n+\tau)]^* \tag{15}$$

However, following the derivation in [16], $\rho_{k,i}$ for polyphase sequences may be well approximated as:

$$\rho_{k,i} \approx 2N^2 \sum_{n=1-N}^{N-1} | c_{k,i}(n) |^2 \tag{16}$$

In order to evaluate the performance of a whole set of $M$ spreading sequences, the average mean-square value of cross-correlation for all sequences in the set, denoted by $R_{CC}$, was introduced by Oppermann and Vucetic [8] as a measure of the set cross-correlation performance:

$$R_{CC} = \frac{1}{M(M-1)} \sum_{i=1}^{M} \sum_{\substack{k=1 \\ k \neq i}}^{M} \sum_{\tau=1-N}^{N-1} | c_{i,k}(\tau) |^2 \tag{17}$$

A similar measure, denoted by $R_{AC}$ was introduced in [8] for comparing the auto-correlation performance:

$$R_{AC} = \frac{1}{M} \sum_{i=1}^{M} \sum_{\substack{\tau=1-N \\ \tau \neq 0}}^{N-1} | c_{i,i}(\tau) |^2 \tag{18}$$

The measure defined by (15) allows for comparison of the auto-correlation properties of the set of spreading sequences on the same basis as the cross-correlation properties. It can be used instead of the figures of merit, which have to be calculated for the individual sequences.

For DS CDMA applications we want both parameters $R_{CC}$ and $R_{AC}$ to be as low as possible [8]. Because these parameters characterize the whole sets of spreading sequences, it is convenient to use them as the optimization criteria in design of new sequence sets. In the considered numerical examples, maximum value of aperiodic cross-correlation functions since this parameter is very important when the worst-case scenario is considered. We will then look into the both parameters $R_{CC}$ and $R_{AC}$. Sequence selection criteria, not based on the correlation characteristics, can be envisaged as well.

## 4. Application to Bipolar Sequences

From the implementation point of view, the most important class of spreading sequences are bipolar or bi-phase sequences, where the $\phi_m$; $m = 1, 2, \ldots, N$, can take only two values 0 and $\pi$. This results in the elements on the diagonal of $\mathbf{D}_N$ being equal to either ‘+1’ or ‘-1’. Even for this bipolar case, we can achieve significantly different properties of the sequences defined by the $\mathbf{W}_N$ than those of the original bipolar sequences of the same length.

To find the best possible modifying diagonal matrix $\mathbf{D}_N$ we can do an exhaustive search of all possible bipolar sequences of length $N$, and choose the one, which leads to the best performance of the modified set of sequences. However, this approach is very computationally intensive, and even for a modest values of $N$, e.g. $N = 32$, it is rather impractical. Hence, other search methods, like a random search, must be considered.

By applying a Monte Carlo algorithm [17], [18] to $N = 32$, and looking for a minimum value of the peaks in the aperiodic cross-correlation functions, $C_{\max}$, in 1000 random draws, we have found the sequence:

$$\mathbf{S} = [+ + - - + - + - - + - - - + + + + - - + + + + + + + + + + + ++] \tag{19}$$

where, for the simplicity, ‘+’ and ‘-‘ correspond to ‘+1’ and ‘-1’, respectively.

The sequence **S** leads to the following parameters of the modified set of sequences:

$$\begin{aligned} C_{\max} &= 0.4063 \\ R_{AC} &= 0.8925\,. \\ R_{CC} &= 0.9738 \end{aligned}$$

For the comparison, the corresponding parameters of the original set of Walsh-Hadamard sequences of length $N = 32$ are as follows:

$$\begin{aligned} C_{\max} &= 0.9688 \\ R_{AC} &= 6.5938\,. \\ R_{CC} &= 0.7873 \end{aligned}$$

It is visible, that a significant improvement has been achieved in terms of reducing the value of $C_{\max}$ and $R_{AC}$. This improvement has been offset by a slight increase in the value of $R_{CC}$. The matrix $\mathbf{W}_{32}$ defining the modified sequence set equals to:

$$
\mathbf{W}_{32} = \begin{bmatrix}
\text{++--+-+--+---++++--++++++++++++} \\
\text{+--+++++---+--+-++--+-+-+-+-+-+-} \\
\text{+++++--+-+++-+--+-+-++--++--++--} \\
\text{+-+-++----+----++++++--++--++--+} \\
\text{++---+-+-+--+---+--+----++++----} \\
\text{+--+-------+++-+++---+-++-+--+-+} \\
\text{++++-++--++++-+++-+---++++----++} \\
\text{+-+---++--+-+++-++++-++-+--+-++-} \\
\text{++--+-+-+-+++---+--+++++--------} \\
\text{+--++++++++-++-+++--+-+--+-+-+-+} \\
\text{+++++--++---+-+++-+-++----++--++} \\
\text{+-+-++--++-++++-+++++--+-++--++-} \\
\text{++---+-++-++-++++--+--------++++} \\
\text{+--+----+++---+-++---+-+-+-++-+-} \\
\text{++++-++-+----+--+-+---++--++++--} \\
\text{+-+---++++-+---+++++-++--++-+--+} \\
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\text{+--+----+++---+---+++-+-+-+--+-+} \\
\text{++++-++-+----+---+-+++--++----++} \\
\text{+-+---++++-+---+----+--++--+-++-}
\end{bmatrix} \tag{20}
$$

In Fig. 1, we present the plot of the upper limits for the peak magnitudes of aperiodic cross-correlation functions for the set of Walsh-Hadamard sequences and the set of sequences defined by the matrix $\mathbf{W}_{32}$. These plots illustrate the significant decrease in the peak magnitudes of the cross-correlation functions that can be achieved through the proposed modification. Fig. 2 illustrates a major improvement that can be achieved in auto-correlation properties of the modified sequences compared to the original Walsh-Hadamard sequences. For the modified sequences, a clear peak is present for the perfect sequence alignment with no other significant peaks for any non-zero shift. This is not the case for Walsh-Hadamard sequences. As a result, no additional scrambling would be required for synchronization of a system utilizing sequences defined by the matrix $\mathbf{W}_{32}$.

Because of the nonlinear character of the cost function it is difficult to assess how far the obtained result is from the global minimum without performing the exhaustive search. Calculating the theoretical lower bound for the aperiodic cross-correlation and aperiodic out-of-phase auto-correlation magnitudes can give some insight into this. The best-known bound is due to Welch [19], and states that for any set of $M$ bipolar sequences of length $N$

$$\max\{c_{i,k}, c_{i-oop}\} \geq \sqrt{\frac{M-1}{2NM-M-1}} \tag{21}$$

where $c_{i\text{-}oop}$ denotes the out-of-phase aperiodic autocorrelation value. In the considered case of 32 sequences of length 32 the Welch bound is equal to 0.1261. A more tighter bound was given by Levenshtein [20], and is expressed by:

$$\max\{c_{i,k}, c_{i-oop}\} \geq \sqrt{\frac{(2N^2+1)M-3N^2}{3N^2(MN-1)}} \tag{22}$$

In the considered case, the Levenshtein bound is equal to 0.1410. It must be noted here that both Welch and Levenshtein bounds are derived for sets of bipolar sequences where the condition of orthogonality for perfect synchronization is not imposed. Hence, one can expect that by introducing the orthogonality condition, the lower bound for the aperiodic cross-correlation and aperiodic out-of-phase auto-correlation magnitudes must be significantly lifted.

In Fig. 3 and Fig. 4, we present the simulation results for the 32 channel asynchronous DS CDMA system utilizing pure Walsh-Hadamard sequences and the sequences defined by the matrix $\mathbf{W}_{32}$, respectively. In both cases, we had simulated the same number of 8 randomly chosen simultaneous active users, and transmitted the same number of 524-bit frames in each of the 32 possible channels. The results have been then averaged across the 32 channels. The transmission channel was assumed to be an AWGN channel with an $E_b/N_0$ = 20 dB. It can be seen that not only the average BER drops by almost 50% in the case of the modified sequences, but what is even more significant, the maximum number of errors in the frames from 219 for Walsh-Hadamard to 54 for the modified sequences.

## 5. Application to quadri-phase sequences

From the implementation viewpoint, another important class of spreading sequences are complex valued, quadri-phase sequences. Those four phases being:

$$\phi_1 = 0, \quad \phi_2 = \frac{\pi}{2}, \quad \phi_3 = \pi, \quad \phi_4 = -\frac{\pi}{2}$$

correspond to the sequence elements taking complex values of:

$$s_1 = 1, \quad s_2 = j, \quad s_3 = -1, \quad s_4 = -j\,,$$

respectively.

To find the appropriate values of phase coefficients $\phi_m$; $m$ = 1, 2, …, $N$, (see Eq. (10)), for $N$ = 32, we used the random search, as in the case of bipolar sequences. Again, we searched for the minimum value of the maximum peak magnitudes in the aperiodic cross-correlation functions, $C_{\max}$, using Monte Carlo approach. The minimum value of $C_{\max}$, achieved after 1000 trials, was 0.3658 for the vector **P** of phase coefficients

$$\begin{aligned}\mathbf{P} &= [\phi_m ;\ m = 1, \ldots, 32] \\ &= [3\ 2\ 3\ 3\ 2\ 2\ 1\ 1\ 1\ 4\ 4\ 3\ 4\ 3\ 3\ 1\ 4\ 3\ 2\ 3\ 1\ 3\ 3\ 2\ 4\ 2\ 2\ 2\ 3\ 3\ 1]\end{aligned} \quad (23)$$

where the numbers 1, 2, 3, and 4, denote the phases 0, $\pi/2$, $\pi$, and $-\pi/2$, respectively. Thus the diagonal of the matrix $\mathbf{D}_{32}$ equals to:

$$[-1\ j -1 -1\ j\ j 1 1 1 -\ j -\ j -1 -\ j -1 -1 1 -\ j -1\ j -1 1 -1 -1\ j -\ j\ j\ j\ j\ j -1 -1\ \ 1]$$

As in the bipolar case, it is difficult to judge how far the obtained result is from the global minimum without performing the exhaustive search. Contrary to the bipolar case, the theoretical bound given by Levenshtein in [21] for complex valued sequences cannot be used here even as a guideline, since our sequence elements cannot be any complex number but they can take only four values, 1, -1, j, and –j.

The mean square correlation measures of the developed quadri-phase sequence set are:

$$\begin{aligned} R_{CC} &= 0.9708 \\ R_{AC} &= 0.9063 \end{aligned}$$

and in Fig.5 and Fig.6, we present the upper limits for the peak magnitudes of aperiodic cross-correlation functions and aperiodic auto-correlation functions, respectively. It is clearly visible, that the achieved sequences exhibit better properties than the original Walsh-Hdamard sequences.

Because of the reduced value of $C_{\max}$ compared with both original Walsh-Hadamard sequences and the modified bipolar sequences considered in the previous Section, one can expect shorter error bursts in the system utilizing these quadri-phase sequences.

This has been confirmed by simulation of a DS CDMA system as in the previous Section, with the only difference being the spreading sequences used. As expected, the maximum number of errors in 524-bit frames dropped to 28, and we achieved a further reduction in an average BER, reduced to BER = 0.0012. This can be noticed in Fig.7, which shows the distribution of a number of errors in the transmitted frames for the simulated 32 channel system.

## 6. Discussion of BER results

Albeit in theory, the increase in the value of $R_{CC}$ for the modified sequences compared with the original Walsh-Hadamard sequences should lead to an increase in the level of MAI [3], this has not been the case observed in the simulation for neither bipolar nor quadri-phase sequences. To contrary, we experienced a drop of around 50% in the average BER. Such a behaviour is caused by the fact that in our simulation we assumed the carrier phases and relative time shifts staying constant for all users during the transmission of a single 524-bit frame. The results obtained in [3] assume that the phases and delays are randomly changed for every transmitted bit. However, the simulated scenario reflects more accurately the realistic situation where users remain continuously active for at least duration of one frame. This means that the relative delays and the carriers' phases are not randomly changed for every single bit but remain constant for the whole frame. In such a case, the very high value of $C_{max}$ = 0.9688 for Walsh-Hadamard sequences causes a very high number of errors, i.e. up to 219 of erroneous bits, in some of the received frames. Such large number of errors in some frames has an adverse impact on the average BER, as has been observed.

In addition, if one considers application of a forward-error-control (FEC) mechanism for real time services, minimization of the number of errors per received frame seems even more important than achieving a low BER with some of received frames being error free and others having a very high number of errors. In such cases, improvement of a worst case MAI reflected in the value of $C_{\max}$ is much more important than improving of an average MAI reflected by the $R_{CC}$.

## 7. Conclusions

In [7] we presented a simple method to modify orthogonal spreading sequences to improve their correlation properties for asynchronous applications, while maintaining their orthogonality for perfect synchronization. The method leads, in general, to the complex polyphase sequences but can also be used to obtain real bipolar sequences. In this paper, we showed that the method could be successfully applied to design orthogonal bipolar or quadri-phase sequences exhibiting good properties in case of asynchronous DS CDMA operation with the set of Walsh-Hadamard sequences of a given length being a starting point. The presented examples indicate that significant improvements can be achieved in transmission quality if, instead of the original Walsh-Hadamard sequences, the sequences designed using the proposed method are employed. In addition, because of much better auto-correlation characteristics, an additional scrambling might not be required for synchronization purposes.

In both considered cases, i.e. bipolar and quadri-phase sequences, there are only limited numbers of modifications possible for a given sequence length. Unfortunately, examining all of them becomes impractical even for a modest sequence length, e.g. $N = 32$. In the paper, we used Monte Carlo technique to find the appropriate modifications. However, other, more advanced search methods may produce even better designs.

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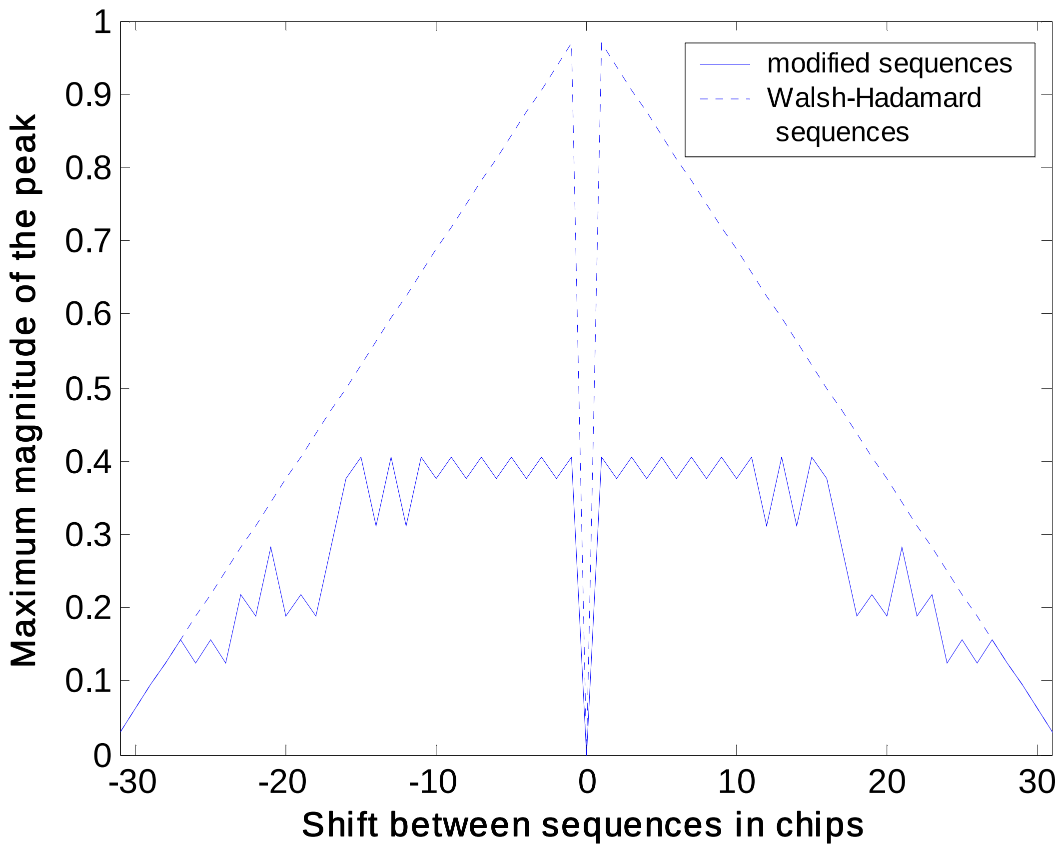


**Figure 1:** Maximum magnitudes of peaks in the cross-correlation functions for the whole sets of spreading sequences of length 32; Walsh-Hadamard sequences – dotted line, modified bipolar sequences- solid line.

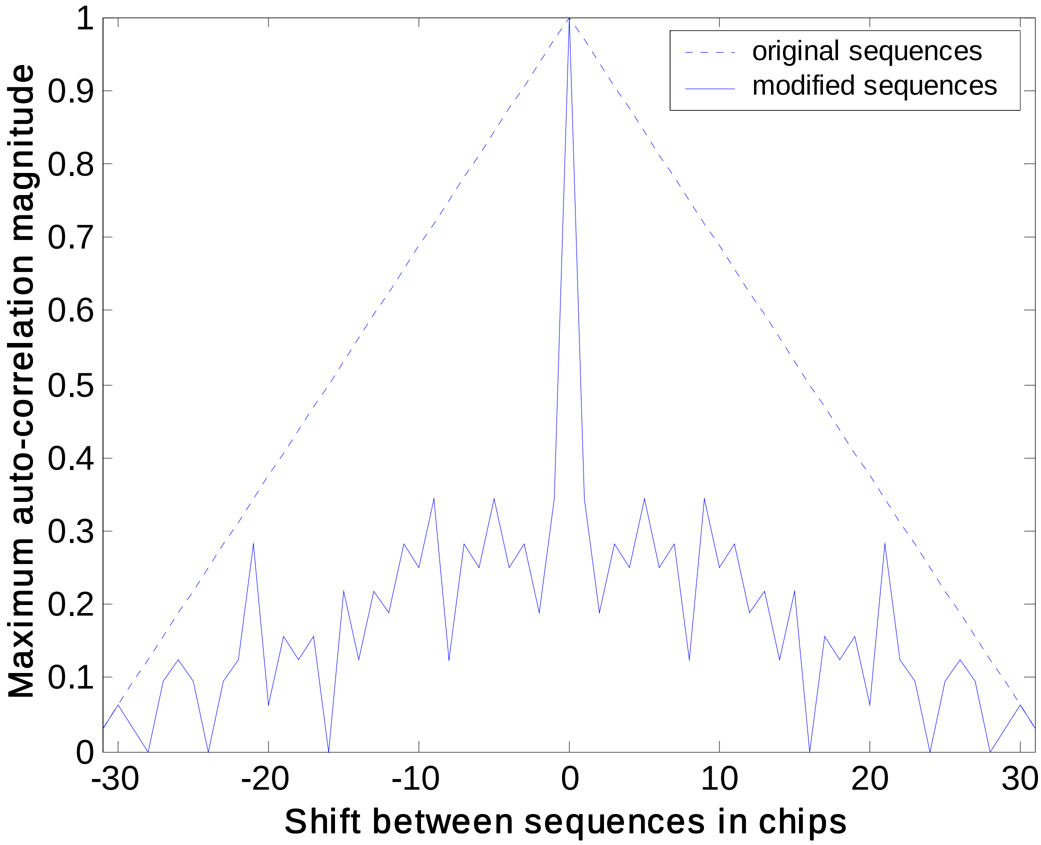


**Figure 2:** Maximum magnitudes of peaks in the auto-correlation functions for the whole sets of spreading sequences of length 32; Walsh-Hadamard sequences – dotted line, modified bipolar sequences- solid line.

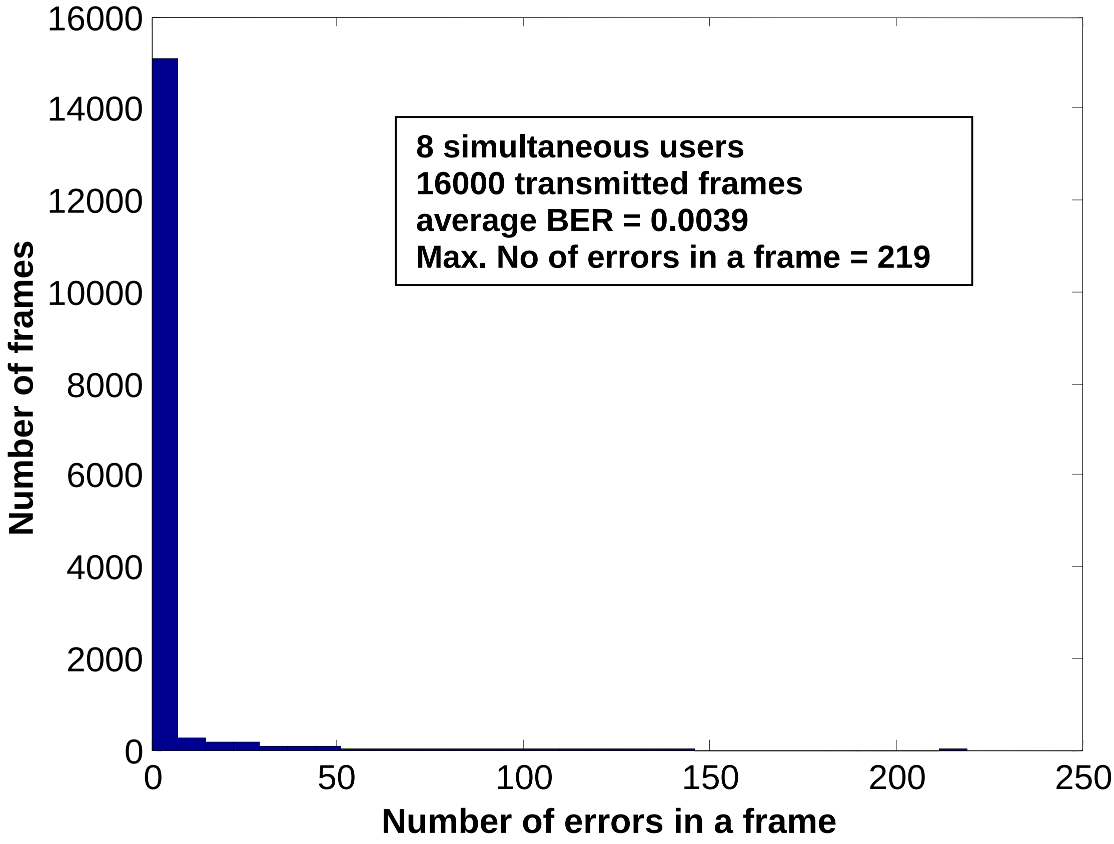


**Figure 3:** Histogram of a number of errors in transmitted frames for the DS CDMA system utilizing Walsh-Hadamard spreading sequences.

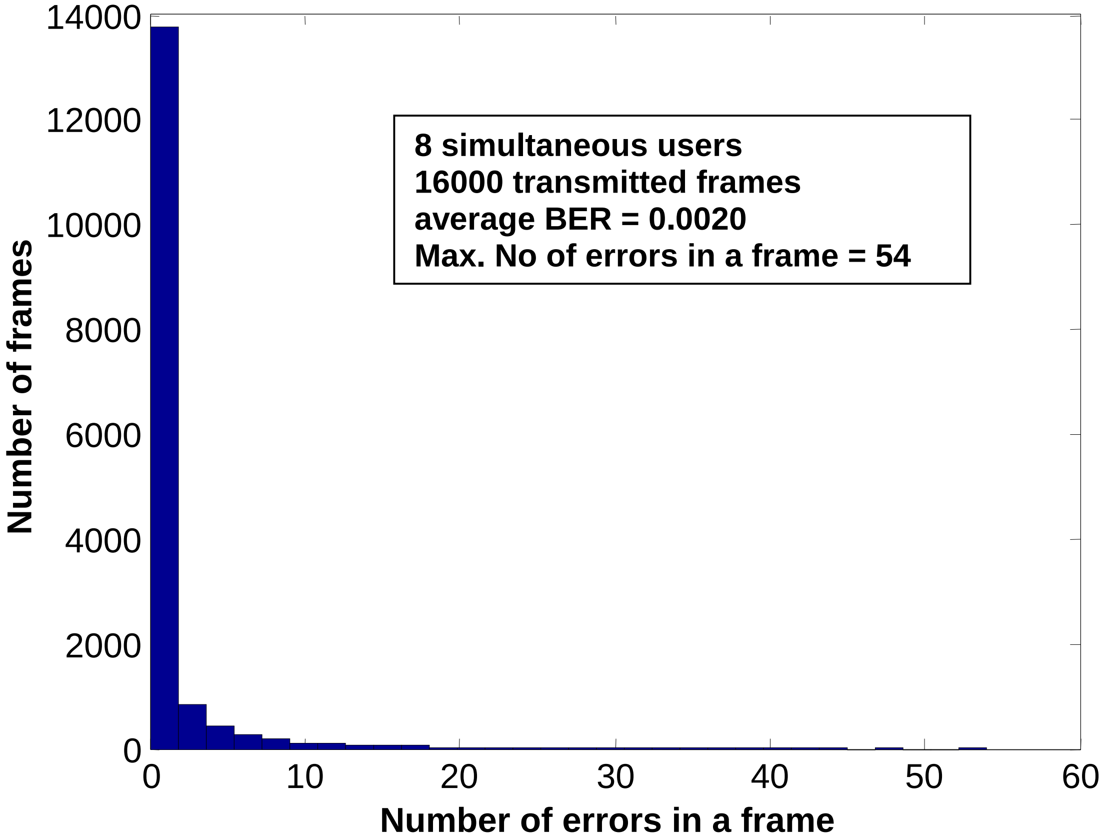


**Figure 4:** Histogram of a number of errors in transmitted frames for the DS CDMA system utilizing spreading sequences defined by the matrix $\mathbf{W}_{32}$.

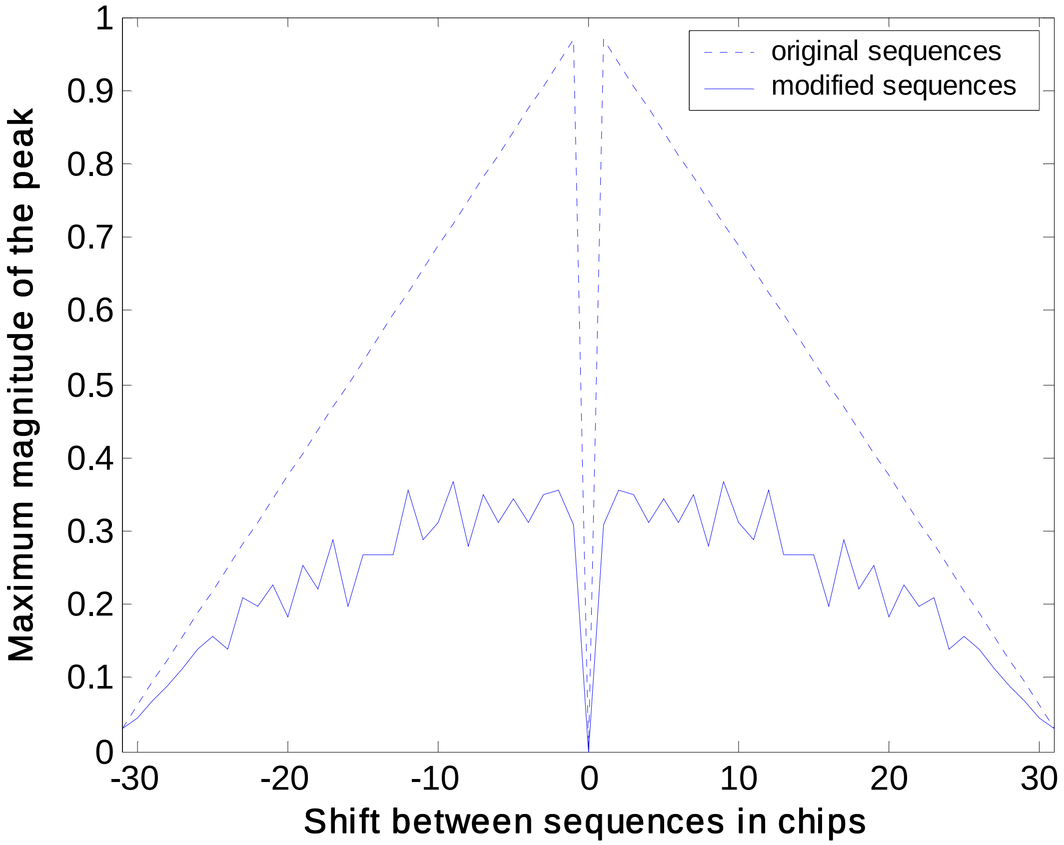


**Figure 5:** Maximum magnitudes of peaks in the cross-correlation functions for the whole sets of spreading sequences of length 32; Walsh-Hadamard sequences – dotted line, modified quadri-phase sequences- solid line.

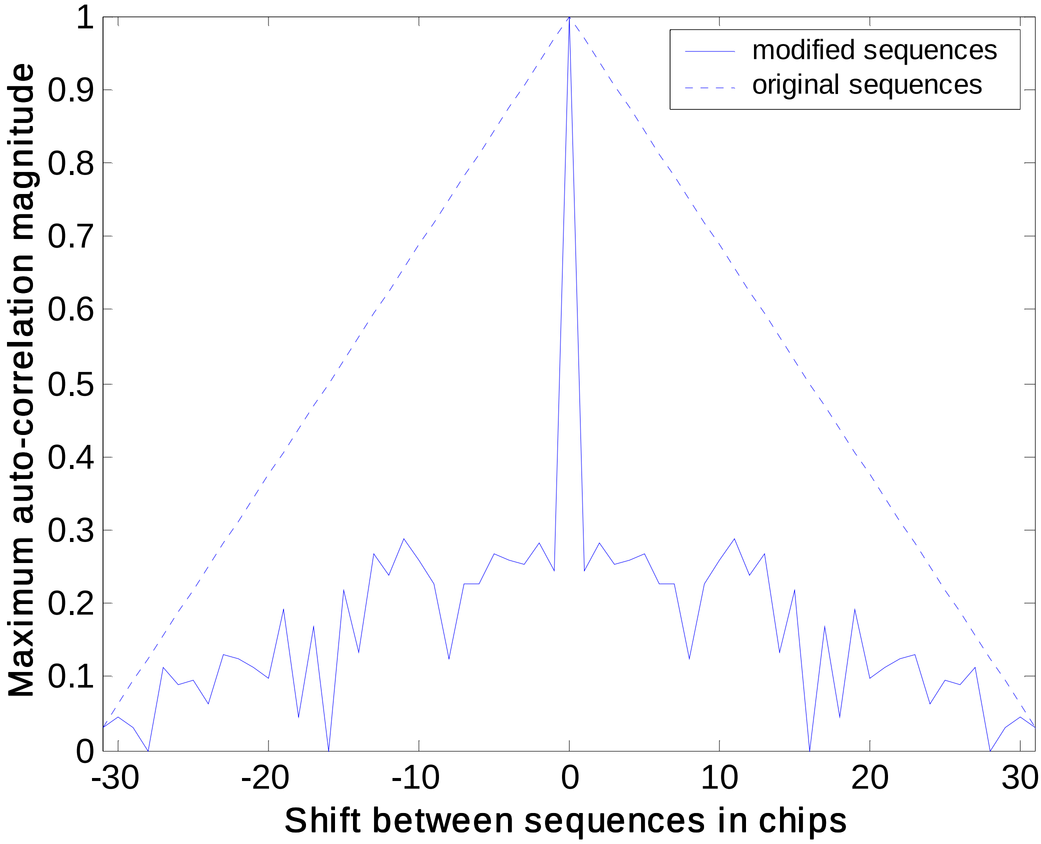


**Figure 6:** Maximum magnitudes of peaks in the auto-correlation functions for the whole sets of spreading sequences of length 32; Walsh-Hadamard sequences – dotted line, modified quadri-phase sequences - solid line.

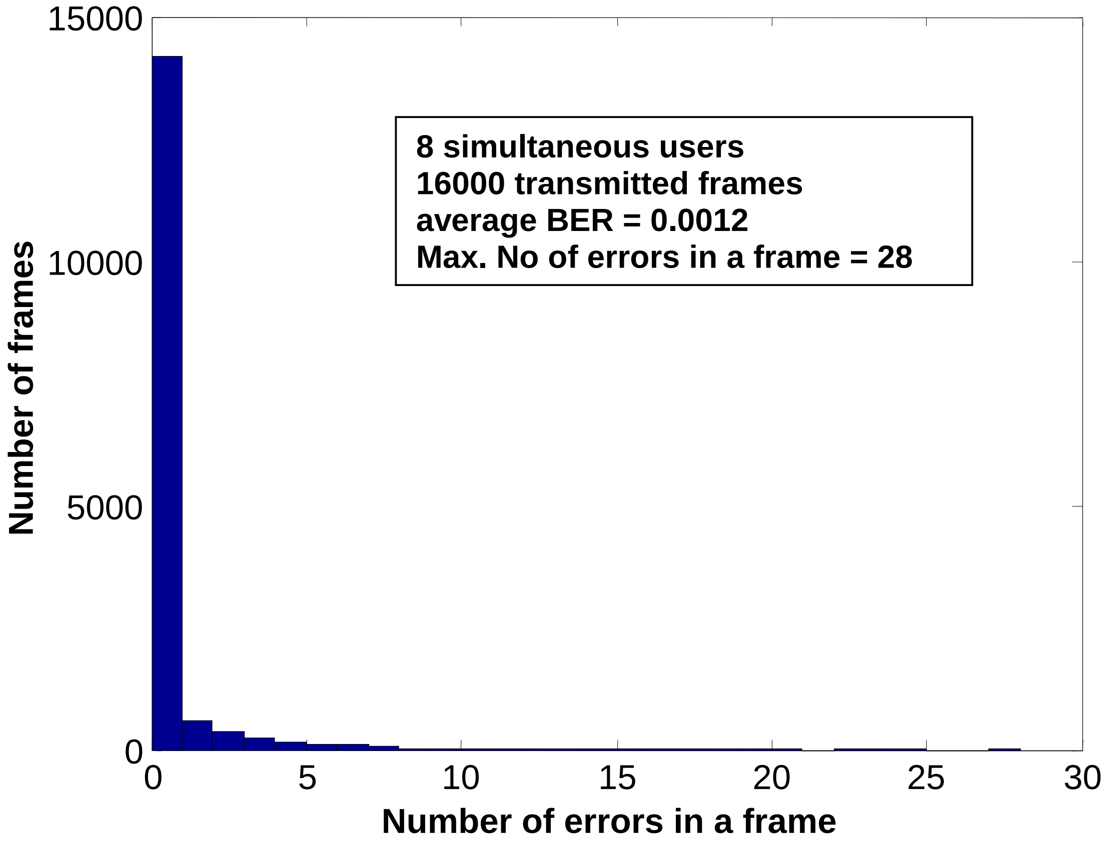


**Figure 7:** Histogram of a number of errors in transmitted frames for the DS CDMA system utilizing modified quadri-phase spreading sequences.