

## CREATING OF DISCRETE POWER SPECTRA FOR FSK

Kittel, L., FernUniversität Hagen, FRG

Slominski, M., Warsaw University of Technology, Poland

Wysocki jr., T., Institute of Telecomm. ATR Bydgoszcz, Poland

Conditions are formulated which could allow the FSK signal to have in its power spectrum a nonzero discrete component which can be utilized for synchronization. The methods investigated are preencoding and preencoding with unbalancing, respectively. Examples are given for MSK where the preencoding without and with unbalancing is performed by two different 3B4B line codes.

### 1. INTRODUCTION

In designing of a communication system, one of the problems is the choice of a modulation technique. In selecting the optimum technique, some key conditions must be satisfied as are: proper spectral characteristics, immunity from noise, and synchronization between transmitter and receiver.

For radio communications, narrow band angular modulations are often used. The spectral properties for the most of them are usually discussed under the assumption that the modulating signal consists of statistically independent data.

To satisfy the spectral requirements for a modulated signal, two methods are generally utilized:

- (i) pulse shaping of the modulating signal, e.g. SFSK, GMSK [1],
- (ii) preencoding combined with modulation [2], [5].

The objective of this paper is to formulate conditions for a preencoding which could allow the FSK signal to have a nonzero discrete component in its power spectrum.

### 2. SPECTRAL ANALYSIS

#### 2.1. A model of the FSK modulated signal

Generally, the FSK modulated signal is an isochronous pulse stream. Thus, the FSK modulator can be treated as a conventional Moore's machine  $M = \langle E, S, G, \delta, \lambda \rangle$ , where  $E$  is a set of data words,  $S$  is a set of states,  $G$  is a set of modulated words (signal pulse sequences),  $\delta: E \times S \rightarrow S$  is a transition function, and  $\lambda: S \rightarrow G$  is an output function.

On the basis of this model, it is easy to find a Markov chain specified by the triple

$$(S, P_{SS}, P_S), \quad (1)$$

where  $S$  is the set of states corresponding to the states of the modulator  $M$ ,  $P_{SS}$  is the transition probability matrix, and  $P_S = [p_1, \dots, p_L]$  is the vector of stationary state probabilities. With regard to the model  $M$ , the FSK modulated signal, denoted by  $x(t)$ , is given by

$$x(t) = \sum_n g^{(n)}(t - nT_w), \quad (2)$$

where  $T_w$  is a word time slot and  $g^{(n)} = \lambda[s^{(n)}]$ .

#### 2.2. Method of analysis

Since the FSK signal is described by the Markov model (1), the modulated signal  $x(t)$  can be expressed by

$$x(t) = \sum_n \bar{g}(t - nT_w) + \sum_n h^{(n)}(t - nT_w), \quad (3)$$

where

$$\bar{g}(t) = E\{g(t)\} = \sum_{k=1}^L p_k \lambda(s_k),$$

and

$$h_k(t) = \lambda(s_k) - \bar{g}(t).$$

The first sum of (3) describes the cyclical signal which is responsible for the appearance of a discrete component in the power spectrum of the modulated signal according to the formula [3]:

$$S_d = \frac{2\pi}{T_w^2} |\bar{G}(\omega)|^2 \sum_r \delta(\omega - \frac{2\pi r}{T_w}). \quad (4)$$

The second sum of (3) corresponds to the continuous power spectrum of the modulated signal which is described by [3]:

$$S_c(\omega) = \frac{1}{T_w} [P_h H^* + C(\omega)], \quad (5)$$

where

$$C(\omega) = 2\text{Re} \left[ \sum_{k=1}^{\infty} P_h (P_{SS})^k H^* \exp(j\omega k T_w) \right].$$

The expressions in (5) have the following meaning:

$\bar{G}(\omega)$  is the Fourier transform of  $\bar{g}(t)$ ,  
 $\delta(\cdot)$  is the Dirac delta function,  
 $P_h = [p_1 H_1(\omega), \dots, p_L H_L(\omega)]$ ,  
 $H = [H_1(\omega), \dots, H_L(\omega)]$ ,  
 $H_i(\omega)$  is the Fourier transform of  $h_i(t)$ , and  
 $H^*$  is the vector conjugated with  $H$ .

### 2.3. Application to a binary FSK

For a binary FSK with angular frequencies  $\omega_0$  and  $\omega_1$ , the elements of the set  $B$  of the modulated signal pulses are given by

$$b_0^i(t - iT) = A \cdot h(t - iT) \cos(\omega_0(t - iT) + \varphi_0^i), \quad (6)$$

$$b_1^i(t - iT) = A \cdot h(t - iT) \cos(\omega_1(t - iT) + \varphi_1^i),$$

where

$$h(t) = \begin{cases} 1 & \text{for } 0 \leq t < T, \\ 0 & \text{otherwise.} \end{cases}$$

For the rational values of a modulation index,  $m = (\omega_1 - \omega_0)T/2\pi$ , the set  $B$  is a finite one. If a modulating signal consists of statistically independent pulses, the initial values of phase angle  $\varphi_0^i$  and  $\varphi_1^i$  have a uniform distribution on the interval  $<0, 2\pi>$ . Therefore,

$$\sum_{\varphi_0^i} p_i \cos(\varphi_0^i) = 0, \quad (7)$$

$$\sum_{\varphi_1^i} p_i \cos(\varphi_1^i) = 0,$$

where  $p_i$  is the probability of a pulse with initial phase  $\varphi_0^i$  or  $\varphi_1^i$  appearance in the modulated signal, respectively. Thus, independent of the probabilities  $p$  and  $q$  of mark and space in the modulating signal, the average pulse of a modulated signal is equal to zero for all values of  $t$ . This indicates the absence of a discrete component in the power spectrum.

## 3. PREENCODED FSK

### 3.1 Condition for discrete power spectrum

As an example of preencoding of modulating data, a block code is considered. For codewords of length  $M$ , an average  $\bar{g}(t)$  of modulated words (of the same length  $M$ ) is given by the expression:

$$\begin{aligned} \bar{g}(t) &= \sum_{i=1}^L p_i \lambda(s_i) = \sum_{i=1}^L p_i g_i(t), \\ &= \sum_{i=1}^L p_i \sum_{j=1}^M b_{ij} [t - (j-1)T], \\ &= \sum_{j=1}^M \sum_{i=1}^L p_i b_{ij} [t - (j-1)T], \end{aligned} \quad (8)$$

where  $L$  is the power of modulated words set, and  $T$  is a single pulse time slot.

From (5) it follows that

$$\begin{aligned} b_{ij}(t) &\equiv 0, \quad \text{for } t \leq 0 \text{ or } t > T, \\ b_{ij}(t) &\neq 0, \quad \text{for } 0 < t \leq T. \end{aligned}$$

Thus, the average value of modulated words is identically equal to zero. The absence of a discrete component in power spectrum occurs if and only if, for each  $j = 1, \dots, M$ ,

$$\sum_{i=1}^L p_i b_{ij} [t - (j-1)T] \equiv 0. \quad (9)$$

A nonzero discrete power spectrum component appears only in the case when, for any  $j = 1, \dots, M$ , the condition (9) is not satisfied.

It follows that such  $j = j^*$  exists for which

$$\sum_{\varphi_0^*} p_i \cos(\varphi_0^*) \neq 0, \quad \text{or} \quad \sum_{\varphi_1^*} p_i \cos(\varphi_1^*) \neq 0, \quad (10)$$

where  $\varphi_0^*$  and  $\varphi_1^*$  are the initial phases of the pulses  $b_{ij^*}(t)$  corresponding to "0" and "1", respectively.

To satisfy the condition (10), line codes or error protection codes can be used as a preencoding scheme.

### 3.2 MSK combined with binary line encoding

To give an example of preencoding schemes for MSK, two different 3B4B codes will be analyzed. Spectral properties of the modulated signals will be discussed under the following assumptions:

- (i) the signal to be encoded consists of random equiprobable binary pulses,
- (ii) the initial value of a phase angle is equal to zero for the first modulated pulse.

(a) MSK combined with 3B4B [4].

The encoding rule of the 3B4B code is given in Tab. 1. The distribution of phases  $\varphi_0^i$  and  $\varphi_1^i$  of the modulated signal is presented for this code in Tab. 2.

000	0101
001	1001
110	0110
111	1010
010	1110   0001
011	1101   0010
100	0111   1000
101	1011   0100

Table 1: The 3B4B [4] mapping

000	0001	0111
001	0011	
010	0101	
011	0110	
100	1001	
101	1010	
110	1100	
111	1000	1110

Table 3: The 3B4B [6] mapping

$\varphi$	Codeword phase								Total	
	1		2		3		4		0	1
	0	1	0	1	0	1	0	1		
0	4	4	0	0	4	4	0	0	8	8
$0.5\pi$	0	0	4	4	0	0	4	4	8	8
$\pi$	4	4	0	0	4	4	0	0	8	8
$1.5\pi$	0	0	4	4	0	0	4	4	8	8

Table 4: Distribution of modulated pulses initial phases for MSK combined with 3B4B [6]

$\varphi$	Codeword phase								Total	
	1		2		3		4		0	1
	0	1	0	1	0	1	0	1		
0	3	5	0	0	3	5	0	0	6	10
$0.5\pi$	0	0	3	3	0	0	3	3	6	6
$\pi$	5	3	0	0	5	3	0	0	10	6
$1.5\pi$	0	0	5	5	0	0	5	5	10	10

Table 2 : Distribution of modulated pulses initial phases for MSK combined with 3B4B [4]

It can be easily verified here by inspection of Tab.2 that condition (10) is satisfied. Therefore, a nonzero discrete component in power spectrum is expected. Results of computations, carried out on the formulae (3) and (4), are plotted in Fig. 1.

(b) MSK combined with 3B4B [6]

The encoding rule of the 3B4B code is given in Tab.3. For this code, the distribution of phases  $\varphi_0^i$  and  $\varphi_1^i$  of the modulated signal is presented in Tab. 4.

By inspection of Tab.4, it can be easily verified that the condition (10) is not satisfied here. Thus, in contrast with the technique (a), a nonzero discrete component does not exist in the power spectrum of the modulated signal. Results of power spectra computations are presented in Fig. 2.

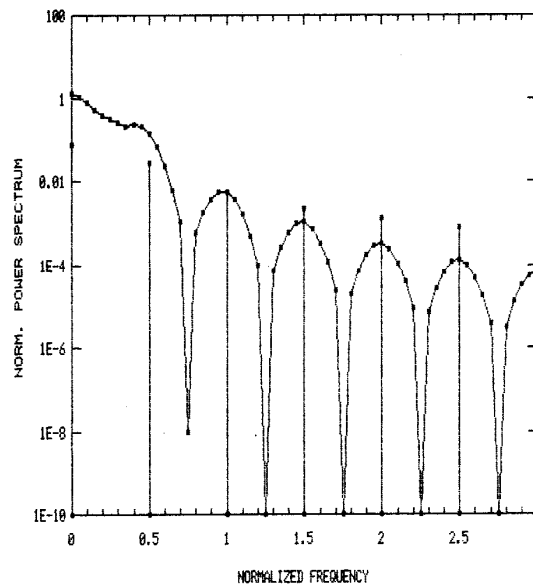


Figure 1: Presence of a nonzero discrete component in power spectrum of MSK combined with 3B4B [4].

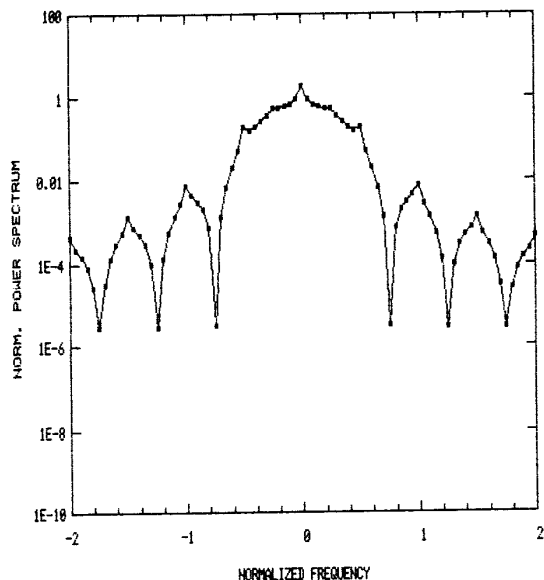


Figure 2: Absence of a nonzero discrete component in power spectrum of MSK combined with 3B4B [6].

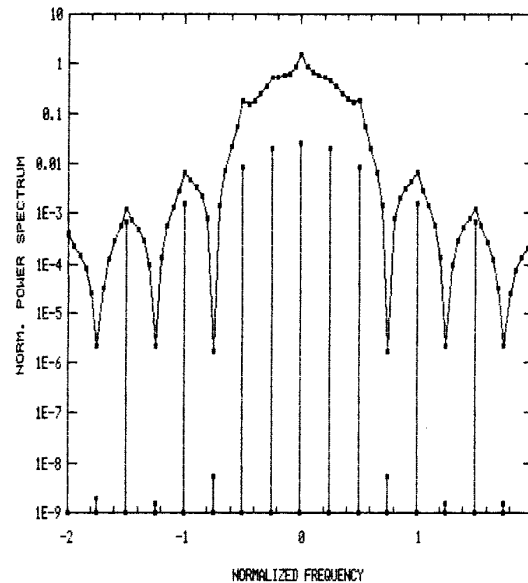


Figure 3: Presence of a nonzero discrete component in power spectrum of MSK combined with 3B4B [6]; unbalanced modulating signal. ( $q = 0.4$ ;  $p = 0.6$ )

Continuing this example, it can be interesting to discuss spectral properties of the modulated signal for a modulating signal being unbalanced, i.e. when the probability of mark differs from the probability of space. For this case, the spectral analysis was done. The results (see Fig.3) confirmed that the unbalanced data stream has caused a nonzero discrete component in the power spectrum of the modulated signal.

#### 4. CONCLUSION

In the paper, a method of creating discrete power spectra for FSK signals is presented. It is based on changing the initial values of modulated pulse phase angle distribution (uniformity violation). To achieve such distributions, preencoding of the modulating data was proposed. Another method discussed was preencoding of modulating data in combination with unbalancing.

The presence of a nonzero discrete component can be utilized for extracting a carrier frequency at the receiver. The method of creating discrete power spectra for FSK presented in this paper can be applied to a synchronous communication system, for which the untreated modulated signal - without preencoding - has the zero discrete power spectrum component.

#### References

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